Dimensionality and viscosity exponent in shear-driven jamming—Supplemental Material

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(Dated: January 22, 2019)

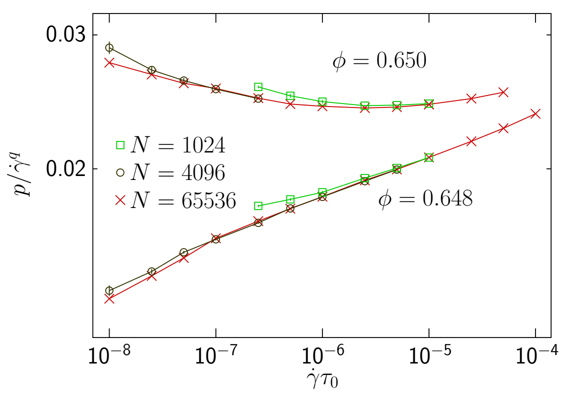

FIG. S1. Finite size study. Pressure vs shear strain rate at densities $\phi = 0.648$ and $\phi = 0.650$, around $\phi_J \approx 0.6491$ for the three different numbers of particles $N = 1024$, $N = 4096$, and $N = 65536$. To make the small differences visible the data are shown as $p/\dot\gamma^q$ with $q = 0.233$, to get a quantity that is almost constant. This comparison gives at hand that finite size effects for $N = 65536$ should be negligible down to $\dot\gamma\tau_0 = 10^{-8}$.

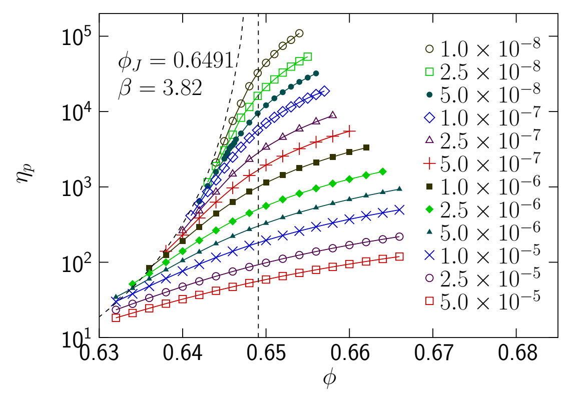

FIG. S2. Data used in the scaling analyses as $\eta_p \equiv p/(\dot\gamma k_d)$. These data are for parameters that obey the conditions $0.632 \leq \phi \leq 0.666$ and $|X| < 0.2$, where $X = (\phi - 0.6491)/\dot\gamma^{0.205}$. Also shown is the divergence $\sim (\phi_J - \phi)^{-\beta}$ with β and ϕ_J from the scaling analysis with $\dot\gamma_{\max}\tau_0 = 10^{-5}$. Some of the results from scaling analyses with different $\dot\gamma_{\max}$ are shown in Fig. 2.

I. FINITE SIZE CHECK

Since our interest is in the behavior of large (infinite) systems it is important to check that the finite size effects in our system with $N = 65536$ particles are negligible. To that end we have run some simulations with two different smaller number of particles, $N = 1024$ and $N = 4096$, to see at which shear rates finite size effects set in for these N. The data, obtained at $\phi = 0.648$ and $\phi = 0.650$, around $\phi_J \approx 0.6491$, are shown as $p/\dot\gamma^q$ in Fig. S1. The finite size effect becomes visible at low shear rates only, and for our two system sizes we find that the data for $N = 1024$ and $N = 4096$ appear reliable down to $\dot\gamma\tau_0 \approx 10^{-5}$ and $\dot\gamma\tau_0 \approx 10^{-7}$, respectively. Though it is not obvious how to extrapolate such results to $N = 65536$, it appears clear that our data for $N = 65536$ should not be at all affected by the finite size effects, down to our lowest $\dot\gamma\tau_0 = 10^{-8}$.

II. SCALING ANALYSES OF PRESSURE

The point with this section is to give some details on the scaling analyses, presented in the Letter.

Fig. S2 shows the raw data as $\eta_p \equiv p/(\dot\gamma k_d)$ vs ϕ for shear rates $\dot\gamma\tau_0 = 10^{-8}$ through 5×10^{-5}. The data are shown for several different shear strain rates, and approach the dashed line, given by $\sim (\phi_J - \phi)^{-\beta}$, as $\dot\gamma \to 0$.

For a simple scaling analysis without corrections to scaling, the value of ϕ_J and two exponents would be enough to make the data collapse to a common curve. In the present case, where corrections to scaling cannot be neglected, it follows from Eq. (5) that it becomes necessary to subtract off the correction term to obtain a data collapse. This data collapse is shown in Fig. S3(a). One way to assess the quality of the fit is by measuring

$$\frac{\chi^2}{\mathrm{DOF}} = \frac{1}{N_{\mathrm{pts}} - N_{\mathrm{par}}} \sum_{i=1}^{N_{\mathrm{pts}}} \left(\frac{p(\phi_i, \dot\gamma_i) - f_{\mathrm{scale}}(\phi_i, \dot\gamma_i)}{\delta p(\phi_i, \dot\gamma_i)} \right)^2, \tag{S1}$$

where N_{pts} is the number of data points, N_{par} is the number of free parameters in the fit (which is here $N_{\mathrm{par}} = 15$), $f_{\mathrm{scale}}(\phi_i, \dot\gamma_i)$ is the right hand side of Eq. (5), and $\delta p(\phi_i, \dot\gamma_i)$ is the estimated statistical uncertainty. Our fit with $\dot\gamma_{\max} = 10^{-5}$ has $\chi^2/\mathrm{DOF} = 1.34$.

Another way examine the fit is by looking at the deviations from the scaling function. However, since these deviations are much too small to be visible in Fig. S3(a), panel (b) instead show the residuals, the quantity inside the big parenthesis in Eq. (S1). If the fit is good the residuals should be independent random variables; clear correlations in the residuals would instead be signs of a questionable fit. Fig. S3(b) shows these residuals, shifted vertically according to shear rate, to improve the visibil-

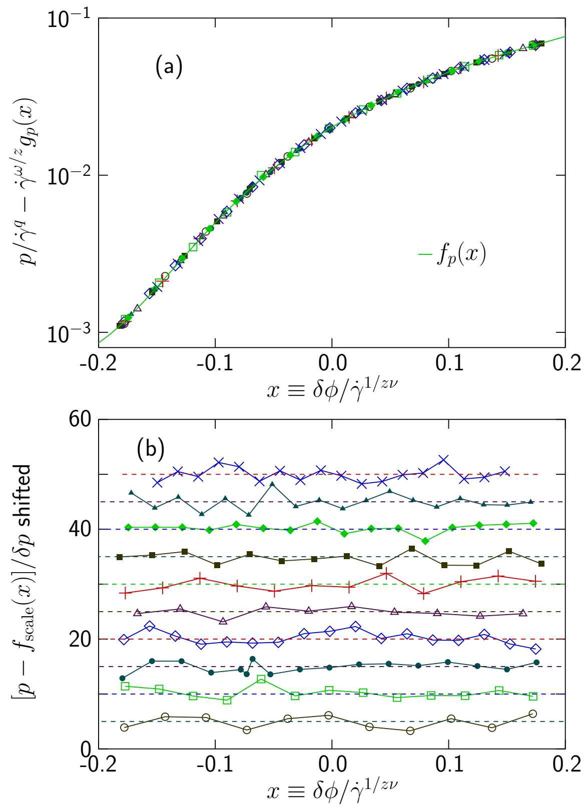

FIG. S3. Scaling collapse after fitting to Eq. (5). Panel (a) shows the collapse of the data, plotted as $p/\dot\gamma^q - \dot\gamma^{\omega/z} g_p(x)$, on the function $f_p(x)$, where $x \equiv \delta\phi/\dot\gamma^{1/z\nu}$. Panel (b) shows the residuals, normalized by the estimated statistical uncertainties, shifted vertically, according to shear rate, for better visibility. The points spread around the scaling function without any clear correlations between points at neighboring densities, as expected for a good fit of the data to the scaling function.

ity. The absence of any clear correlations between the residuals in these curves, suggest that the data do indeed fit well to the scaling function.