Dissipation and Rheology of Sheared Soft-Core Frictionless Disks
Supplemental Material

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RELATIVE MOTION OF CONTACTS

To characterize the nature of the shear-induced particle collisions in our soft-core models, we consider a quantity that we call the angle of contact θ. We define this as the angle that the velocity difference $\mathbf{v}_i - \mathbf{v}_j$ makes with respect to the particle separation $\mathbf{r}_i - \mathbf{r}_j$ for two particles in contact. In Fig. 1 we show plots of the histogram $\mathcal{P}(\theta)$ of the angle of contact θ for our different collisional models. Figs. 1a, b, c are for models CD, $\mathrm{CD_n}$ and $\mathrm{CD_t}$ respectively, in the strongly inelastic case of $Q = 0.1$. Fig. 1d is for model CD in the weakly inelastic case of $Q = 10$. We see that for all models, for the denser values of $\phi \gtrsim 0.6$, $\mathcal{P}(\theta)$ shows a strong peak at $\theta = -90°$, i.e. we have primarily tangential relative motion at contacts. There is essentially no normal relative motion at $\theta = 0$, except at low ϕ.

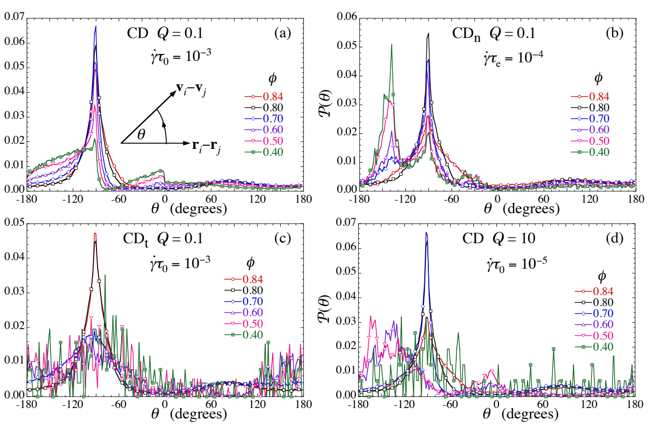

FIG. 1. (color online) Histograms of the angle of contact θ for (a) CD, (b) $\mathrm{CD_n}$, and (c) $\mathrm{CD_t}$ at $Q = 0.1$, and (d) CD at $Q = 10$. Only every fifth symbol is plotted for clarity. Inset to (a) shows the definition of θ.

RHEOLOGICAL CURVES

In Fig. 1 of the main paper we presented plots of the dimensionless elastic part of the pressure $P^{\rm el}$ vs an approriate dimensionless strain rate $\dot\gamma\tau_0$ or $\dot\gamma\tau_e$. The choice of τ_0 was used for systems with overdamped Newtonian rheology at small Q, while τ_e was used for systems with inertial Bagnoldian rheology at small Q, so that the data for small Q collapses to a common curve in each case. It is interesting to look at such rheology curves but now plotted with the opposite choice for dimensionless strain rate, i.e. where in Fig. 1 of the main paper we had plotted vs $\dot\gamma\tau_0$, here we plot vs $\dot\gamma\tau_e$, and vice versa. We show such plots in Figs. 2 below. As should be expected, we now no longer see any simplifying data collapses at any values of Q.

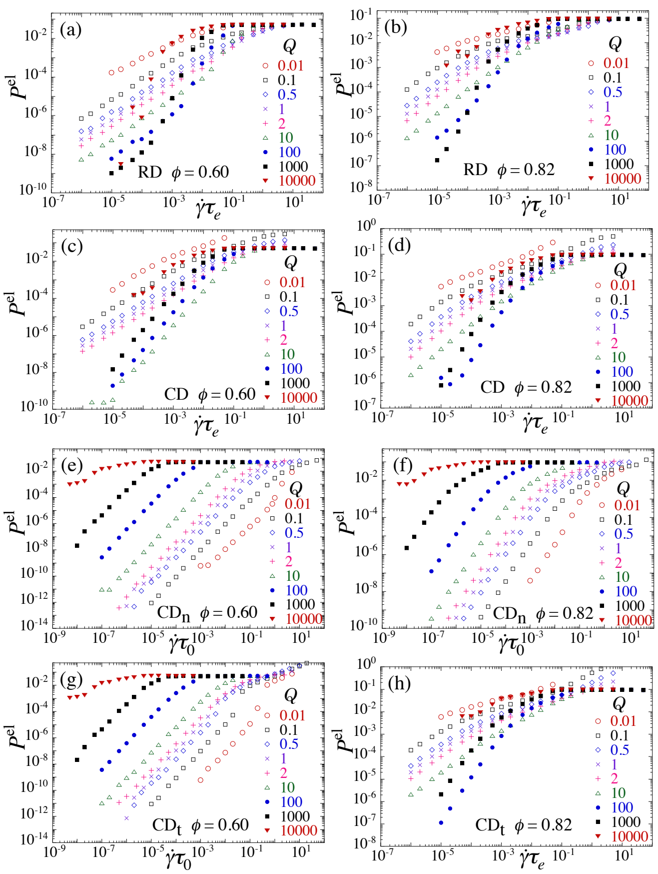

FIG. 2. (color online) Dimensionless pressure $P^{\rm el}$ vs dimensionless strain rate, $\dot\gamma\tau_0$ or $\dot\gamma\tau_e$, for different values of Q for the four dissipative models defined in the main paper. Left hand column is for packing fraction $\phi = 0.60$; right hand is for $\phi = 0.82$, close below jamming. For each value of Q, several different choices of m_s and k_d were used.

PARTICLE CLUSTERING IN MODEL RD

Our discussion of the relation between rheology and clustering, as shown in Fig. 2 of the main article, was limited to the collisional dissipation CD-models. Here we discuss the situation for the reservoir dissipation model RD.

Newtonian rheology results whenever the dissipative term dominates over the kinetic term. For the CD-models, where energy dissipation is due to binary particle collisions, the strength of the dissipative term depends on how long a given collision lasts (collision time is short when particles separate after colliding, collision time is long when particles stick together after colliding). For RD, a particle's energy dissipation is with respect to the uniform sheared background, with which the particle is *always* in contact. Hence the dissipative term never becomes negligible and we always have Newtonian rheology at small $\dot{\gamma}$.

The presence of Newtonian rheology in RD is thus not necessarily related to particle clustering as it is for the CD-models. Nevertheless we can still ask how the average contact number z and the percolation probability f_p vary with ϕ for model RD. We show these quantities in Fig. 3 below, for several different strain rates $\dot{\gamma}$ for the overdamped case of $Q = 0.1$. We see that the contact number z stays finite for all ϕ, with no strong dependence on $\dot{\gamma}$. Thus particles tend to remain in contact with other particles, unlike the case when one has Bagnold scaling where $z \to 0$ as $\dot{\gamma} \to 0$. The percolation probability stays roughly equal to unity above $\phi \approx 0.6$, but then drops rapidly to zero below. Thus at low ϕ the particles are in clusters, but the clusters do not percolate across the system. We do not yet understand if this percolation transition in RD at $\phi \approx 0.6$ has any physical consequences; it does not appear to effect the rheology.

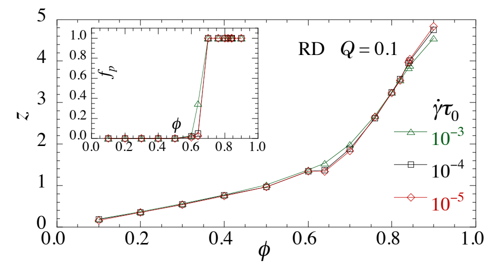

FIG. 3. (color online) Average contact number z vs ϕ at different strain rates $\dot{\gamma}$ for model RD in the overdamped limit, $Q = 0.1$. The inset shows the fraction of states f_p with percolating connected clusters.