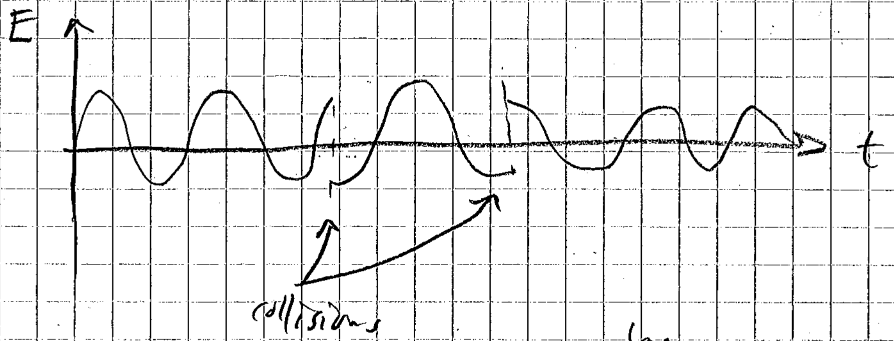
3.1
3.3
(3.4)
(3.5)
(3.7)
(3.8)

5. Classical coherence theory

Consider an atom that is radiating light



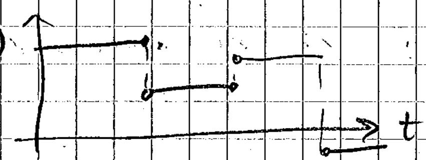
Simple model: collisions $\Rightarrow$ random phase shifts

Average time τ_0 between coll.

[Realistically $\omega_0 \tau_0 \sim 10^5$]

Write $E(t) = E_0 e^{-i\omega_0 t + i\varphi(t)}$ for one atom

where $\varphi(t)$

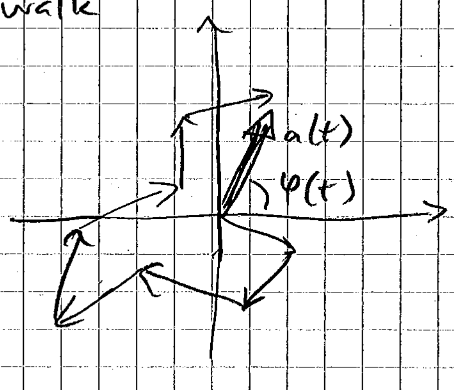


(Simplify: Assume identical E, λ one mode)

At a fixed t, field from N atoms

$$E(t) = E_1(t) + \dots + E_N(t)$$
$$= E_0 e^{i\omega_0 t} \underbrace{\left\{ e^{i\varphi_1(t)} + e^{i\varphi_2(t)} + \dots + e^{i\varphi_N(t)} \right\}}_{\equiv a(t) e^{i\varphi(t)}}$$

random walk

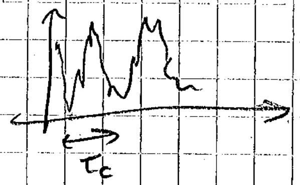


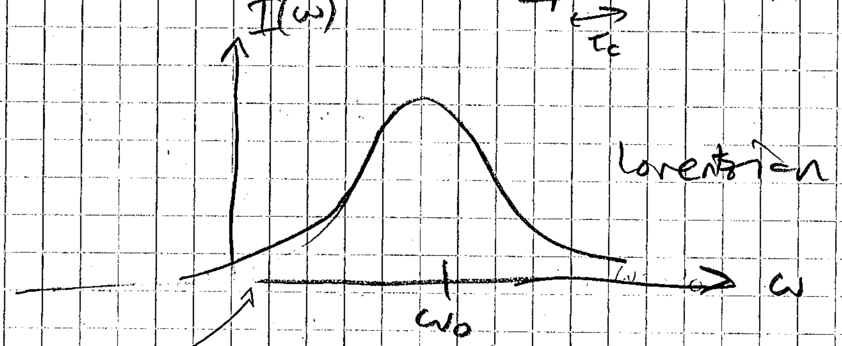
5-2

Average over one period $\frac{2\pi}{\omega_0}$ to get irradiance

$$I(t) = \frac{1}{2}\varepsilon_0 c |E(t)|^2 = \frac{1}{2}\varepsilon_0 c E_0^2 a^2(t)$$

$I(t)$ example see Fig. 3.4

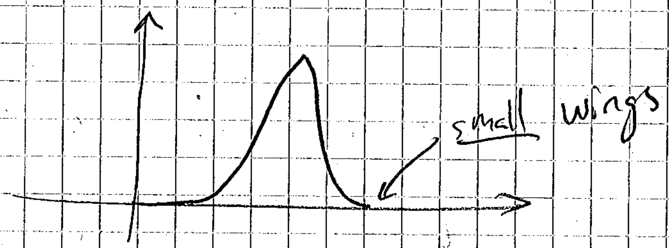




wide wings contain many frequencies

$\tau_c = \frac{1}{\gamma_{coll}}$

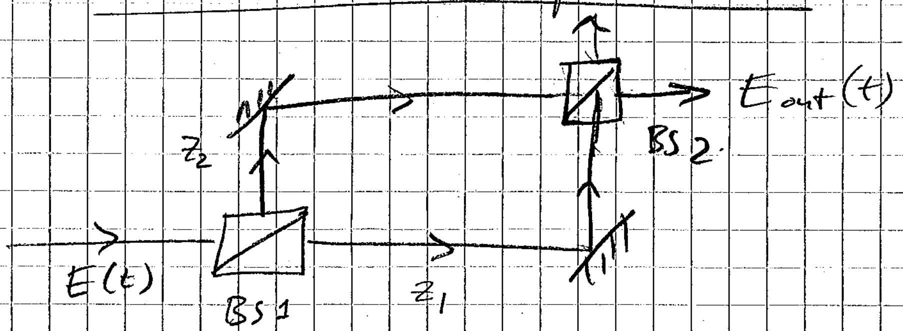
Assume instead only Doppler broadening → Gaussian



fluctuations are smoother.

$\tau_c = \frac{\sqrt{\pi}}{\Delta}$, Δ is width of Gaussian

Mach-Zehnder interferometer



Each beam-splitter has reflection coefficient R
transmission — " — T

Arm 1: Transmitted in first
path length z_1
Reflected in second

$$E_{out}(t) = R\,T\,E\left(t - \frac{z_1}{c}\right) + T\,R\,E\left(t - \frac{z_2}{c}\right)$$

Path lengths translates to time difference
let $\tau = \frac{z_2 - z_1}{c}$

$$I_{out} = \frac{1}{2}\varepsilon_0 c^2 \left\langle |E_{out}|^2 \right\rangle$$

$$= \frac{1}{2}\varepsilon_0 c^2 \Big\langle |R|^2 |T|^2 |E(t)|^2 + |R|^2 |T|^2 |E(t+\tau)|^2 + |R|^2 |T|^2\, 2\, \mathrm{Re}\left(E^*(t) E(t+\tau)\right) \Big\rangle$$

Study the cross-term

$$\langle E^*(t) E(t+\tau) \rangle = \frac{1}{T}\int_0^T dt\, E^*(t) E(t+\tau)$$

Define the degree of first-order coherence

$$\boxed{g^{(1)}(t) = \frac{\langle E^*(t) E(t+\tau) \rangle}{\langle E^*(t) E(t) \rangle}}$$

5-4

$$\Rightarrow I_{out} = 2|R|^2|T|^2 I_{in}\left\{1 + \operatorname{Re} g^{(1)}(\tau)\right\}$$

Note $g^{(1)}(\tau) = g^{(1)}(-\tau)^*$

Mach-Zehnder fringe pattern $\Leftarrow$ degree of 1-order coh.

Calculate for one atom · Chaotic light:

$$\langle E^*(t)E(t+\tau)\rangle = \left\langle E_0 e^{i\omega_0 t} e^{i\varphi_1(t)} E_0 e^{-i\omega_0(t+\tau)} e^{-i\varphi_1(t+\tau)}\right\rangle$$

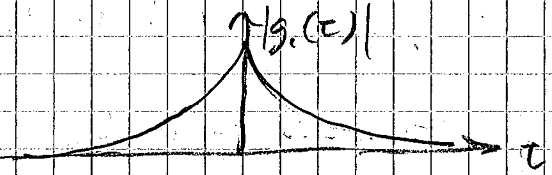
$$= E_0^2 e^{-i\omega_0\tau} \underbrace{\left\langle e^{i(\varphi_1(t)-\varphi_1(t+\tau))}\right\rangle}_{=\begin{cases}1 & \text{if no collision in time } \tau\\ \text{random} & \text{if collision in time } \tau\end{cases}}$$

$= \text{Prob(No collisions in time } \tau)$

$= \int_\tau^\infty d\tau' \, p(\tau')$ with $p(\tau') = \frac{1}{\tau_0} e^{-\frac{\tau'}{\tau_0}}$

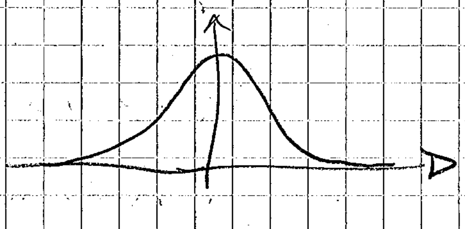
$= e^{-\tau/\tau_0}$

$$\therefore g^{(1)}(\tau) = e^{-i\omega_0\tau - \frac{|\tau|}{\tau_0}} = e^{-i\omega_0\tau - \gamma_{coll}|\tau|}$$



Doppler-broadened:

similarly $g^{(1)}(\tau) = e^{-i\omega_0\tau - \frac{\pi}{2}\left(\frac{\tau}{\tau_c}\right)^2}$



Correlation for Doppler-broadened light

Write $$E(t) = E_0 \sum_{i=1}^{N} \underbrace{e^{-i\omega_i t + i\varphi_i}}_{\text{contribution from each atom}}$$

$$\langle E^*(t) E(t+\tau) \rangle = E_0^2 \sum_{i,j} \left\langle e^{i\omega_i t} e^{-i\varphi_i} e^{-i\omega_j (t+\tau)} e^{i\varphi_j} \right\rangle$$

$$= E_0^2 \sum_{i,j} \left\langle e^{i(\omega_i - \omega_j)t} e^{-i\omega_j \tau} \underbrace{e^{i\varphi_j - i\varphi_i}}_{=0} \right\rangle$$

if $i \neq j$!

$$= E_0^2 \sum_{i=1}^{N} e^{-i\omega_i \tau}$$

{Maxwell-Boltzmann}

$$= N E_0^2 \int d\omega \, P(\omega) \, e^{-i\omega\tau}$$

$$= N E_0^2 \frac{1}{\sqrt{2\pi\Delta^2}} \int d\omega \, e^{-\frac{(\omega_0 - \omega)^2}{2\Delta^2}} e^{-i\omega\tau}$$

{where $\Delta = \omega\sqrt{k_B T / mc^2}$}

$$= N E_0^2 e^{-i\omega_0 \tau - \frac{1}{2}\Delta^2 \tau^2}$$

$$\therefore \quad g^{1}(\tau) = e^{-i\omega_0 \tau - \frac{\pi}{2}\left(\frac{\tau}{\tau_c}\right)^2} \qquad \tau_c = \frac{\sqrt{\pi}}{\Delta}$$

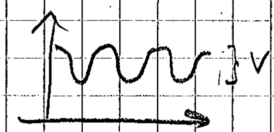
$g^{(1)}(\tau)$ describes the correlation between the field at two different times (and/or points) since $\tau \to \Delta t - \frac{\Delta x}{c}$ gives the same result

We say that

$$|g^{(1)}(\tau)| = \begin{cases} =1 & \text{first-order coherent} \\ =0 & \text{incoherent} \\ <1, >0 & \text{partially coherent} \end{cases}$$

- Visibility for interference fringes

def. $$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \mathrm{Re}\, g^{(1)}(\tau)$$



- Power spectrum.

Frequency space $E_T(\omega) = \frac{1}{\sqrt{2\pi}} \int_0^T dt\, E(t)\, e^{i\omega t}$

Observe that the power spectrum

$$f(\omega) = \frac{|E_T(\omega)|^2}{T} = \frac{1}{2\pi T} \int_0^T dt \int_0^T dt'\, E^*(t)\, E(t')\, e^{-i\omega(t-t')}$$

$$= \frac{1}{2\pi} \int_0^T d\tau \langle E^*(t)\, E(t+\tau) \rangle e^{i\omega\tau}$$

$$\approx \text{''} \int_{-\infty}^{\infty} \text{''}$$

Define normalized spectrum by dividing by $\langle |E|^2 \rangle$

$$F(\omega) = \frac{|E_T(\omega)|^2 / T}{\langle |E(t)|^2 \rangle} = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\tau\, g^{(1)}(\tau)\, e^{i\omega\tau}$$

The Wiener-Khintchine theorem

Spectrum $\longleftrightarrow$ fluctuations, coherence

5-6 $g^{(1)}$ - correlation of fields

$g^{(2)}$ - — " — irradiance

The degree of second-order coherence

$$\boxed{g^{(2)}(\tau) = \frac{\langle I(t) I(t+\tau)\rangle}{\bar{I}^2} = \frac{\langle E^*(t)E^*(t+\tau)E(t+\tau)E(t)\rangle}{\langle E^*(t)E(t)\rangle^2}}$$

⇝ measure the irradiance at time t and then at $t+\tau$

- repeat many times.

I is real ⇒ Cauchy's inequality applies

$$\langle I(t)\rangle^2 \leq \langle I^2(t)\rangle$$

$$\Rightarrow \quad 1 \leq g^{(2)}(0) \leq \infty$$ holds at $\underline{\underline{\tau = 0}}$. [Classically!]

It also holds that

$$0 \leq g^{(2)}(\tau) \leq g^{(2)}(0)$$ decreases with time.

N atoms radiating

$$E(t) = \sum_{i=1}^{N} E_i(t)$$

$$\langle E^*(t)E^*(t+\tau)\, E(t+\tau) E(t)\rangle = \sum_{i,j,k,l} \langle E_i^*(t) E_j^*(t+\tau) E_k(t+\tau) E_l(t)\rangle$$

$$= \sum_i \langle E_i^*(t) E_i^*(t+\tau) E_i(t+\tau) E_i(t)\rangle$$

$$\sum_{i\neq j} \langle E_i^*(t) E_j^*(t+\tau) E_j(t+\tau) E_i(t)\rangle + \langle E_i^*(t) E_j^*(t+\tau) E_i(t+\tau) E_j(t)\rangle$$

(other terms average out to zero)

$$= N\langle |E_i(t)|^2 |E_i(t+\tau)|^2\rangle$$

$$+ N(N-1)\langle |E_i(t)|^2\rangle\langle |E_j(t+\tau)|^2\rangle + N(N-1)\langle E_i^*(t) E_i(t+\tau)\rangle \langle E_i(t) E_i^*(t+\tau)\rangle$$

(first term smaller)

$N \ll N(N+1)$

$$\approx N^2\left(\langle |E_i(t)|^2\rangle^2 + |\langle E_i^*(t)E_i(t+\tau)\rangle|^2\right)$$

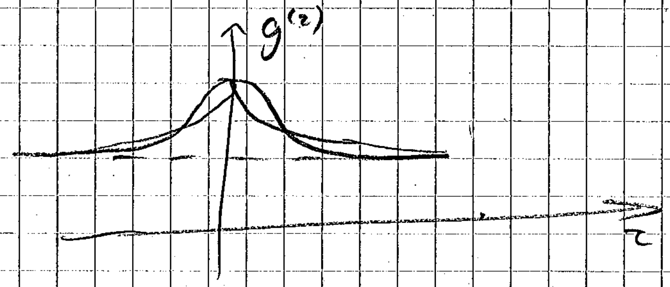
$$g^{(2)}(\tau) = \frac{\langle I(t)I(t+\tau)\rangle}{I^2} =$$

$$= \frac{N^2}{N^2}\left\{\frac{I^2 + |\langle E^*(t)E(t+\tau)\rangle|^2}{I^2}\right\}$$

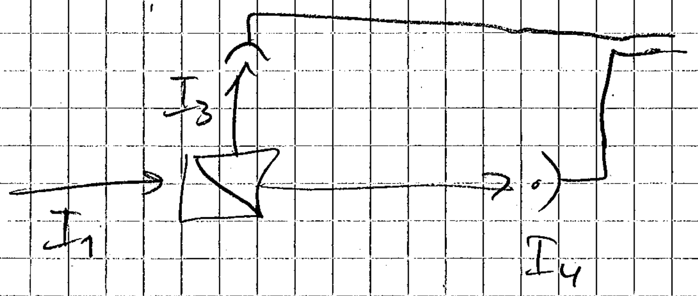
$$\boxed{g^{(2)}(\tau) = 1 + g^{(1)}(\tau)}$$ for chaotic light.

thus: $g^{(2)}(0)=2$

$g^{(2)}(\tau) \to 1$ when $\tau \to \infty$



5-8 The Hanbury Brown-Twiss interferometer



Path length difference $\tau = \frac{1}{c}\Delta z$

$$\langle I_3(t) I_4(t) \rangle = \langle I_1(t) I_1(t+\tau) \rangle$$

$$= I^2 g^{(2)}(\tau) \quad !$$

[This is going to be <u>much more</u> interesting when we do quantum.]