1.6-1.7; 1.9-1.10

(2)

26/3
15-17

<u>When</u> is stimulated emission greater than spontaneous, and vice versa?

Rewrite

$$\langle W_T(\omega)\rangle = \frac{A_{21}}{\frac{g_1}{g_2} e^{\hbar\omega/k_BT} B_{12} - B_{21}} = \frac{A_{21}}{B_{21}} \frac{1}{e^{\hbar\omega/k_BT}-1}$$

$$= \frac{A_{21}\langle n\rangle}{B_{21}} \Rightarrow \frac{B_{21}\langle W_T(\omega)\rangle}{A_{21}} = \langle n\rangle \text{ the ratio.}$$

Stimulated ≈ spontaneous when
We have $\langle n\rangle = 1$ for $\frac{\hbar\omega}{k_BT} \approx 0.7$ (by e.g. Graphical solution)

If $T \approx 300\,K$ then $\omega = 0.7 \cdot \frac{10^{-23}}{10^{-34}} \cdot 300 \sim 3 \cdot 10^{+(34-23)}\, s^{-1} = 3 \cdot 10^{+13}\, s^{-1}$

$\lambda = \frac{2\pi}{k} = \frac{2\pi c}{\omega} \sim \frac{10 \cdot 10^8}{10^{+13}} \approx 10^{-4} \sim 100\,\mu m$ (really $70\,\mu m$)
far-infrared.

Consider the 2 cases

1) $\hbar\omega \ll k_BT \Rightarrow \lambda \gg 70\,\mu m$ microwave, RF

Then $\langle n\rangle \gg 1 \Rightarrow A_{21} \ll B_{21}\langle W_T(\omega)\rangle$

Spontaneous emission can be neglected & the thermal radiation will determine the level population

2) $\hbar\omega \gg k_BT \Rightarrow \lambda \ll 70\,\mu m$

(Visible light, our main concern)

$\langle n\rangle \ll 1 \Rightarrow A_{21} \gg B_{21}\langle W_T(\omega)\rangle$

thermal excitation is negligible

-2

But we can add visible light "by hand"
(expose the sample to laser light)

$$\langle W(\omega)\rangle = \langle W_T(\omega)\rangle + \langle W_E(\omega)\rangle \approx \langle W_E(\omega)\rangle$$

Define $\boxed{W_s \equiv \frac{A_{21}}{B_{21}} = \frac{\hbar\omega^3}{\pi^2 c^3}}$

the saturation energy density.

Saturation irradiance: $I_s = c W_s = \frac{\hbar\omega^3}{\pi^2 c^2}$

s.t. $I > I_s \Rightarrow$ stimulated emission becomes important.

1.7 Optical excitation of 2-level atoms

Assume a non-degenerate 2-level atom

$g_1 = g_2 \Rightarrow B_{12} = B_{21} = B, \quad A_{21} = A$

$$\frac{dN_1}{dt} = N_2 A + (N_2 - N_1) B \langle W \rangle = -\frac{dN_2}{dt}$$

Steady state $\frac{d}{dt} = 0$

$$N_2 A + (N_2 - N_1) B \langle W \rangle = 0 \qquad \left\{ \frac{A}{B} = W_s \right\}$$

$$N_2 W_s + (N_2 - N_1) \langle W \rangle = 0$$

use $N_1 + N_2 = N$

$$N_2 W_s + (2N_2 - N) \langle W \rangle = 0$$

$$\Rightarrow N_2 (W_s + 2\langle W \rangle) = N \langle W \rangle$$

$$\Rightarrow \boxed{N_2 = \frac{\langle W \rangle}{W_s + 2\langle W \rangle} N} = \frac{N B \langle W \rangle}{A + 2B\langle W \rangle}$$

$$\Rightarrow \boxed{N_1 = \frac{W_s + \langle W \rangle}{W_s + 2\langle W \rangle} N}$$

Low irradiance $\langle W \rangle \ll W_s$: $\quad N_1 \approx N$

$$N_2 \approx \frac{W}{\langle W_s \rangle} N$$

High irradiance $\langle W \rangle \gg W_s$: $\quad N_1 \approx \frac{1}{2} N$

$$N_2 \approx \frac{1}{2} N$$

the excitation level <u>saturates</u>, and reaches a value of $N/2$

4

Mean <u>rate</u> of spontaneous emission
(# spont. em. events per unit time)

$$N_2 A = NA \frac{\langle W \rangle}{W_s + 2\langle W \rangle} \rightarrow \frac{1}{2} NA \quad \text{when } \langle W \rangle \to \infty$$

<u>saturates</u>

mean rate of stim. em.

$$N_2 B \langle W \rangle = N \frac{\langle W \rangle}{W_s + 2\langle W \rangle} \frac{A}{W_s} \langle W \rangle \rightarrow \frac{1}{2} NA \frac{\langle W \rangle}{W_s}$$

increases

<u>Time dependence</u>

<u>Ex.</u> Assume that some atoms are in the excited state and then the light is turned off

$$\frac{dN_2}{dt} = -N_2 A$$

$$\Rightarrow N_2(t) = N_2(0) e^{-At} \equiv N_2(0) e^{-t/\tau_R}$$

the population decays exponentially on a characteristic time scale τ_R:

$$\tau_R = \frac{1}{A} = \text{"the radiative lifetime"}$$