The Quantum Zeno effect

The quantum measurement postulate:

Consider an observable $\hat{A}$
with eigenvalues and -states

$$\hat{A}|j\rangle = a_j|j\rangle \qquad j = 0, \dots, \infty$$

Suppose that the state immediately before time t is

$$|\psi(t^-)\rangle = \sum_{j=0}^{\infty} c_j |j\rangle$$

At time t, measure the value of A.
Obtain one of the values a_j, $j = 0, \dots, \infty$
with probability $|c_j|^2$

If the outcome is a_j, then the state becomes

$$|\psi(t^+)\rangle = |j\rangle$$

immediately after measurement.
"Collapse of the wavefunction".

The collapse is an experimental fact
- easiest seen by probing the system again after first measurement.

[Note: this does not follow from the Schrödinger equation.]

"philosophical problem":

Is "measurement" a well-defined concept?

Exactly how & when does collapse take place?

→ through interaction with macroscopic object?

Nobody really knows today...

Quantum measurement for density matrix

A measurement of the state sets the off-diagonal elements (coherences) to zero.

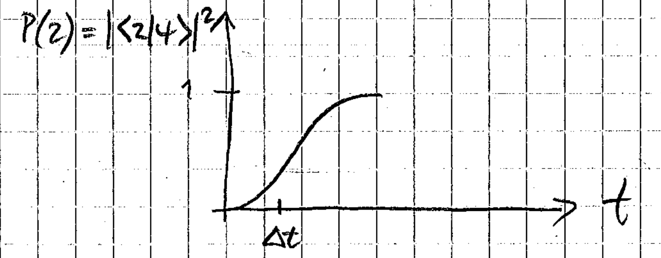
$$\rho(t^-) = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix} \Longrightarrow \rho(t^+) = \begin{pmatrix} \rho_{11} & 0 \\ 0 & \rho_{22} \end{pmatrix}$$

$\rho(t^+)$ describes probabilities - the statistical description.

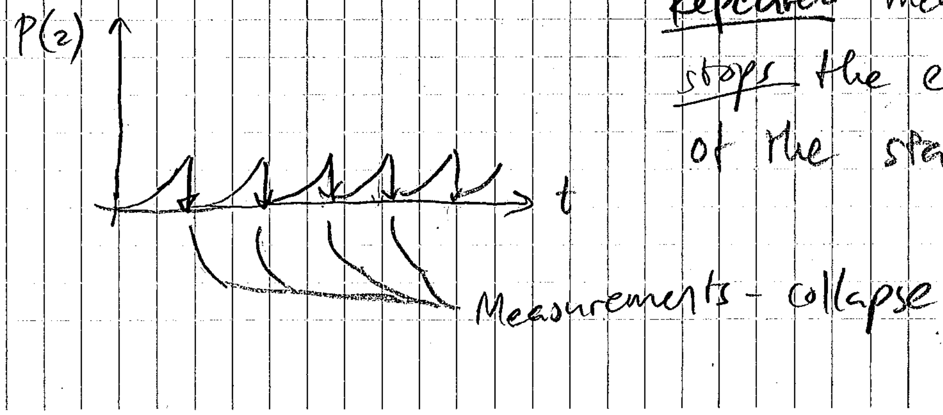
Can be generalized to other observables - by a change of basis.

Quantum Zero effect, general idea

A system starts in state $|1\rangle$ (say) and starts evolving to state $|2\rangle$ if unperturbed



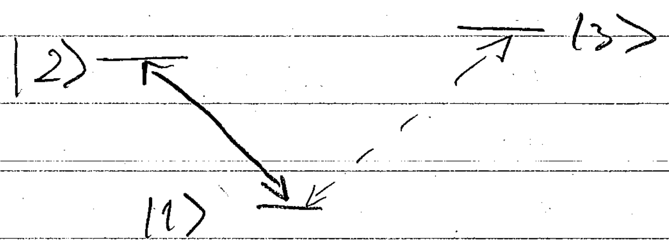
A measurement after a short time Δt collapses the state - with greatest probability to state $|1\rangle$. From that point the evolution starts over again.



Repeated measurements stops the evolution of the state!

Specific example: Experiment by Itano et al., 1992

3-level atom, (ion actually) V configuration



Drive the transition $|1\rangle \leftrightarrow |2\rangle$ so it performs Rabi oscillations. (on resonance)

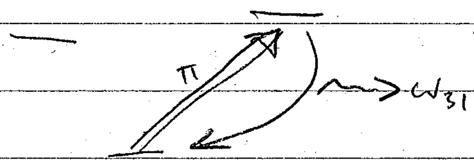
If $\rho_{22}(0)=1$ then [remember!]

$$\rho_{22}(t) = \cos^2 \frac{\Omega t}{2}$$
$$\rho_{11}(t) = \sin^2 \Omega t/2$$

If $\rho_{11}(0)=1$ then
$$\rho_{22} = \sin^2 \frac{\Omega t}{2}$$
$$\rho_{11} = \cos^2 \Omega t/2$$

The state $|3\rangle$ is used to measure whether the ion is in state $|1\rangle$.

Turn on a short π-pulse that takes $|1\rangle$ to $|3\rangle$ if the particle was in $|3\rangle$. Then observe decay photon emitted when $|3\rangle \to |1\rangle$ spontaneously.



This is the measurement.

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The period of one half ~~of a half~~ Rabi oscillation is $T = \frac{\pi}{\Omega}$

Let $\rho_{11}(0) = 1$ or $\rho(0) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

then if the system is left undisturbed (no measurement), then

$$\rho_{11}(T) = 0 \quad \text{or} \quad \rho(T) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

But: Do a measurement at $t = T/2$

then $$\rho\left(\tfrac{T}{2}^{+}\right) = \begin{pmatrix} \cos^2\frac{\Omega T}{4} & 0 \\ 0 & \sin^2\frac{\Omega T}{4} \end{pmatrix} = \begin{pmatrix} \cos^2\frac{\pi}{4} & 0 \\ 0 & \sin^2\frac{\pi}{4} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

Then, the unitary evolution starts over

Write $\rho\left(\frac{T}{2}^{+}\right) = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

& apply known solutions

$$\rho\left(\frac{T}{2} + t\right) = \frac{1}{2}\begin{pmatrix} \cos^2\frac{\Omega t}{2} & \\ & \sin^2\frac{\Omega t}{2} \end{pmatrix} + \frac{1}{2}\begin{pmatrix} \sin^2\frac{\Omega t}{2} & \\ & \cos^2\frac{\Omega t}{2} \end{pmatrix}$$

and at time T,

$$\rho(T) = \frac{1}{2}\begin{pmatrix} \frac{1}{2} & \\ & \frac{1}{2} \end{pmatrix} + \frac{1}{2}\begin{pmatrix} \frac{1}{2} & \\ & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \\ & \frac{1}{2} \end{pmatrix}$$

that is: $\rho_{22}(T)$ = probability of doing transition to $|2\rangle$

$= 1$ in undisturbed case

now it is reduced to $\frac{1}{2}$

because of measurement.

Now do n measurement with spacing $T/n = \Delta t$

$$\rho_{11}(\Delta t) = \cos^2 \frac{\Omega \Delta t}{2} \qquad \rho_{22}(\Delta t) = \sin^2 \frac{\Omega \Delta t}{2}$$

then a measurement

$$\rho_{11}(2\Delta t) = \rho_{11}(\Delta t)\cos^2 \frac{\Omega \Delta t}{2} + \rho_{22}(\Delta t)\sin^2 \frac{\Omega \Delta t}{2}$$

$$= c^2 \cdot c^2 + s^2 \cdot s^2 \qquad \left\{ \text{Let } c = \cos \frac{\Omega \Delta t}{2} \quad s = \sin \frac{\Omega \Delta t}{2} \right\}$$

$$\rho_{22}(2\Delta t) = s^2 \cdot c^2 + c^2 \cdot s^2$$

$$\rho_{11}(j\Delta t) = \rho_{11}((j-1)\Delta t) \cdot c^2 + \left[1 - \rho_{11}((j-1)\Delta t)\right] s^2$$

$$= s^2 + \rho_{11}((j-1)\Delta t)\,(c^2 - s^2)$$

but $s^2 = \frac{1}{2} - \frac{1}{2}\cos \Omega \Delta t$

$c^2 - s^2 = \cos \Omega \Delta t$

so

$$\rho_{11}(n\Delta t) = \frac{1}{2} - \frac{1}{2}\cos \Omega \Delta t + \cos \Omega \Delta t \cdot \left(\frac{1}{2} - \frac{1}{2}\cos \Omega \Delta t + \cos \Omega \Delta t \cdot \left(\frac{1}{2} - \frac{1}{2}\cos \cdots \right.\right.$$

$$\left.\left. \cdots \cos \Omega \Delta t \cdot \left(\frac{1}{2} + \frac{1}{2}\cos \Omega \Delta t \right) \right) \right) \cdots \Big)$$

{cancellations between all terms except last!}

$$= \frac{1}{2} + \frac{1}{2}\cos^n \Omega \Delta t$$

Now, if $\Omega \Delta t$ is small, $\cos \Omega \Delta t = 1 - \frac{1}{2}(\Omega \Delta t)^2$

$$\cos^n \Omega \Delta t = 1 - \frac{n}{2}(\Omega \Delta t)^2$$

$$\rho_{11}(n\Delta t) = \frac{1}{2} + \frac{1}{2}\left(1 - \frac{n}{2}(\Omega \Delta t)^2\right) = 1 - \frac{n}{4}(\Omega \Delta t)^2 \qquad \left\{ \Delta t = \frac{\pi}{n\Omega} \right\}$$

$$= 1 - \frac{n}{4}\left(\frac{\pi}{n}\right)^2 = 1 - \frac{\pi^2}{4n}$$

18-6) Conclusion: Transition probability can be made arbitrarily small by increasing n.

Experiment Itano et al., 1990 → figure 8.11 in Greenstein handout

Note: * Can also start in $|2\rangle$, and transition to $|1\rangle$ inhibited.

→ Then no photon is detected if ion is in $|2\rangle$

⇒ Q.Z.E. results from a sequence of null measurements!

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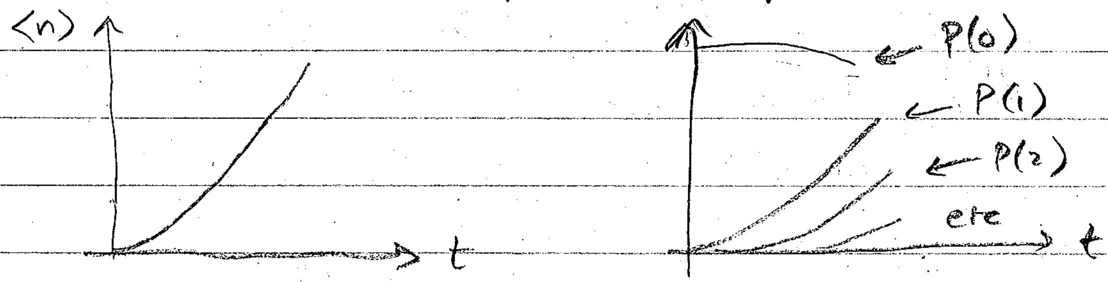
Microwave cavities, QND

Can also realize Quantum Zero effect in a microwave cavity – a maser – where a coherent field is being built up. ↙ See secs. 7.3-7.4 London

Number of photons:



Slow buildup – few photons in a few × 100 ms thanks to ~~very~~ low temperature among others

Measure number of photons → Quantum Zeno effect.

But, cannot just absorb photons to measure them. Too crude!

A quantum non-demolition (QND) measurement scheme for photons

Prepare an atom in a superposition of two states $|50\rangle$ and $|51\rangle$ (highly excited states — "Rydberg states")

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}\left(|50\rangle + |51\rangle\right)$$

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\left(e^{-i\omega_{50}t}|50\rangle + e^{-i\omega_{51}t}|51\rangle\right) \text{ if no light.}$$

Interaction with light will change the relative phase

$$\frac{d}{dt}\hat{\pi} = i\omega_0 - \sum_{k\lambda}\left(2\hat{\pi}^{+}\hat{\pi} - 1\right)\hat{a}_{k\lambda} \quad \& \text{ so on}$$

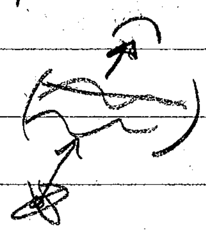
$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\left(e^{-i\omega_{50}t}|50\rangle + e^{-i(\omega_{51}+g_{k\lambda})t}|51\rangle\right) \text{ if one photon.}$$

Now apply a $\frac{\pi}{2}$-pulse to get it to the state $|50\rangle$

→ But: the final state depends on the relative phase of $|50\rangle$ and $|51\rangle$ [See (*)] → page 9

So the final state is $|50\rangle$ if no photon but some superposition $\alpha|50\rangle + \beta|51\rangle$ if one photon!

Sequence:



prepare atom state
Shoot it through the cavity
then $\frac{\pi}{2}$-pulse
then measure state of atom.

If there are no photons | one photon

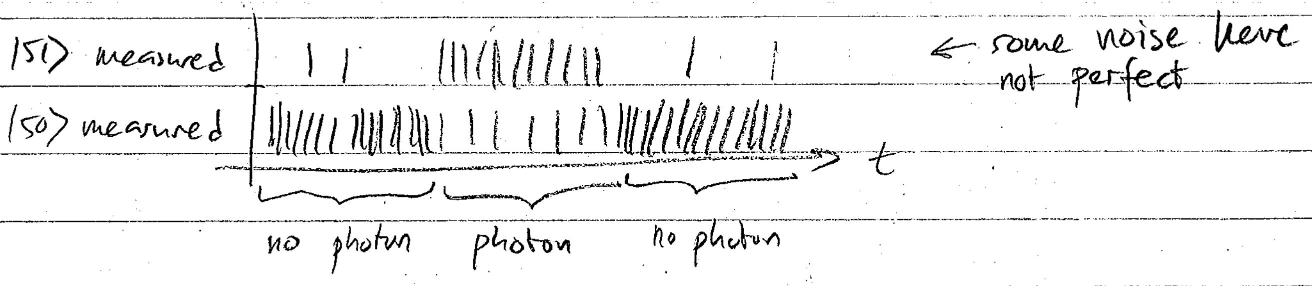
then the relative phase of atom is $e^{i\omega_0 t}$ | $e^{i(\omega_0+g)t}$

then a $\frac{\pi}{2}$-pulse results in $|50\rangle$ | $\alpha|50\rangle + \beta|51\rangle$

Measurement of the state yields $|50\rangle$ all the time | some of the times

and the photon is left intact.

A sequence of atoms can be used to measure a time series.



Generalized scheme allows for measuring the number of photons
(Basically, an "interrogation" method: one atom is set to match the case of one photon, another atom matches two photons, etc.)

(*) $\frac{\pi}{2}$ - pulse

Can easily be derived from OBE:

If $\rho_{11} = \rho_{22} = \frac{1}{2}$

and $\tilde{\rho}_{12} = \tilde{\rho}_{21}^* = \frac{1}{2} e^{i\theta_0}$

Or: $|\psi\rangle = |S0\rangle + e^{-i\omega_0 t + i\theta} |S1\rangle$

then a $\pi/2$ - pulse: Light on for $T = \frac{\pi}{2\Omega}$

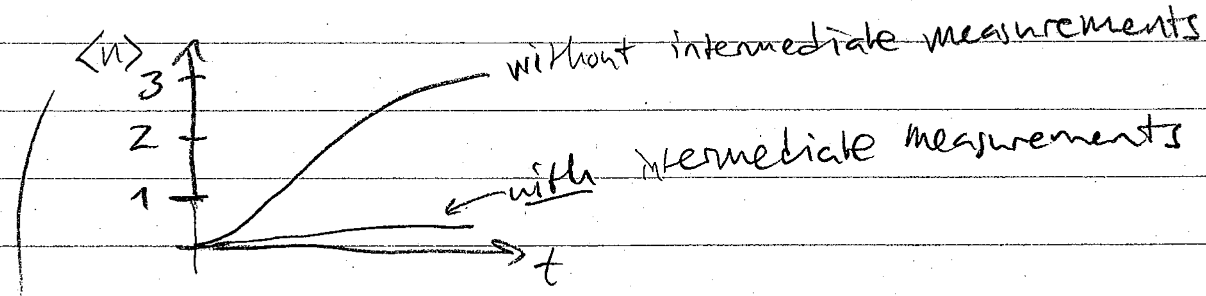
takes the state to $\rho_{11} = \frac{1}{2}(1 - \sin\theta_0)$

$\rho_{22} = \frac{1}{2}(1 + \sin\theta_0)$

Therefore, final state is
$$\begin{cases} \rho_{11} = 1 & \text{if } \theta_0 = -\frac{\pi}{2} \rightarrow |S0\rangle \\ \rho_{22} = 1 & \text{if } \theta_0 = +\frac{\pi}{2} \rightarrow |S1\rangle \\ \text{a superposition otherwise} & \end{cases}$$

This is why a $\pi/2$-pulse can measure the phase of the atomic state.

* Finally, use the QND measurement to create a Quantum Zeno effect!



⟨n⟩ - average is taken over many realizations of same experiment.

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Post scriptum:

(See end of section in Greenstein/Zajonc handout)

Does QZE probe the collapse of the wavefunction?
Or does the experiment scramble the phase of the quantum object in a way that could in principle be predicted by the Schrödinger equation?

- Unclear today...