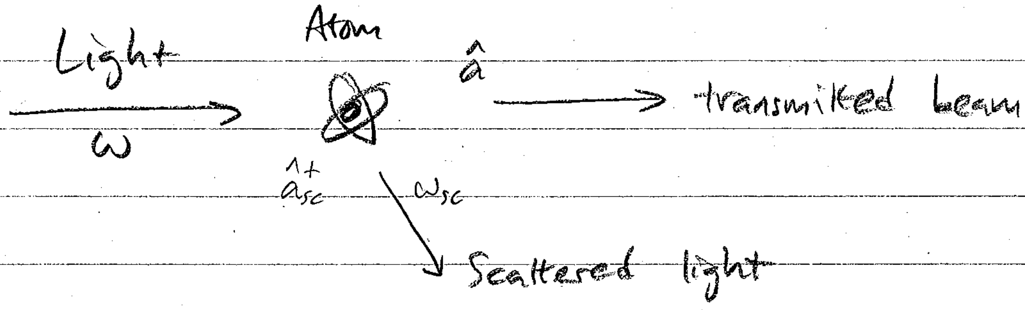
15. Light scattering



Destroy one photon in the incident mode,
and create one in the scattering mode.

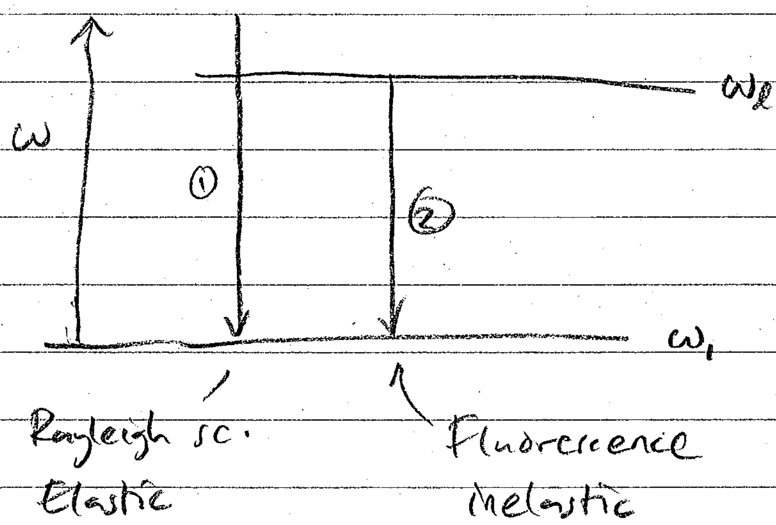
Scattering is greatly enhanced when the incident light is close to resonant, but can happen also off-resonance

The scattering can be

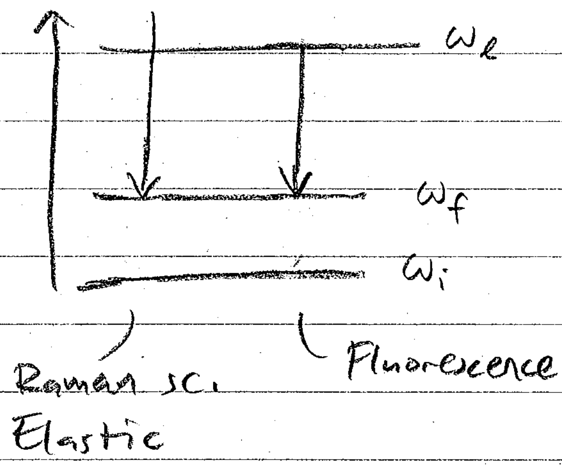
elastic: $\omega_{sc} = \omega$

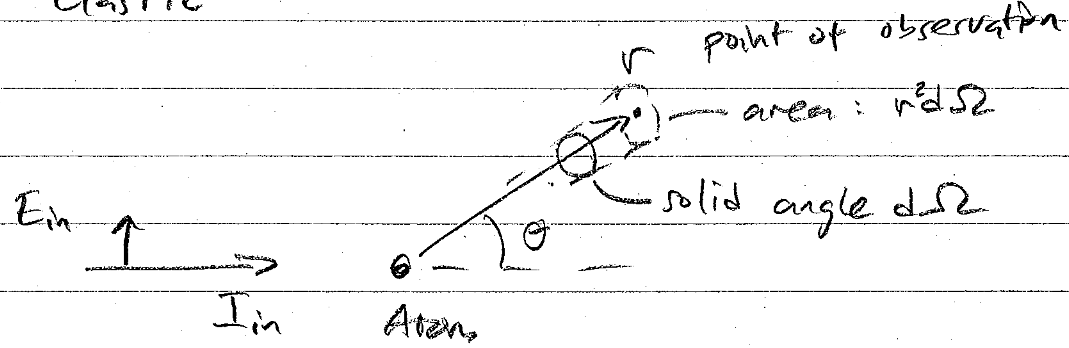
or inelastic: $\omega_{sc} \neq \omega$ ← then the excess energy is taken up by kinetic energy of atom or other deg. of freedom

In a two-level system, have two basic types of process



If there are more levels, can also have





the <u>differential cross-section</u>

def. $$\frac{d\sigma(\omega)}{d\Omega} = r^2 \frac{\omega}{\omega_{sc}} \frac{I_{sc}}{I}$$

the ratio of the flux of photons in the direction (r, θ) to the incident flux.

Incident flux of photons : $\frac{I_{in}}{\hbar\omega_{in}}$

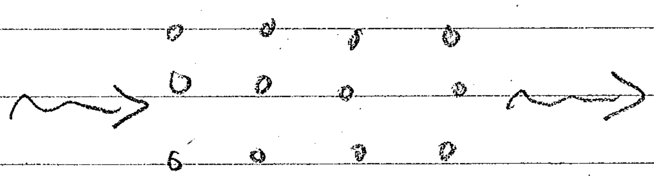
Scattered into $d\Omega$: $\frac{I_{sc}}{\hbar\omega_{sc}} r^2 d\Omega$

[Note: $\frac{d\sigma(\omega)}{d\Omega}$ really independent of r^2, since $I_{sc} \propto \frac{1}{r^2}$ the photons will spread out.]

The total scattering cross-section

$$\sigma(\omega) = \int_{\substack{\text{unit}\\\text{sphere}}} r^2 d\Omega \, \frac{\omega I_{sc}}{\omega_{sc} I_{in}}$$

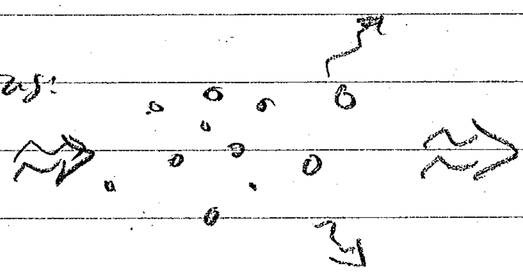
This is for scattering off one atom.
For N atoms, it depends:



1) Perfect crystal   Only the original wave survives.

All scattered light cancels by destructive interference

2) Gas:

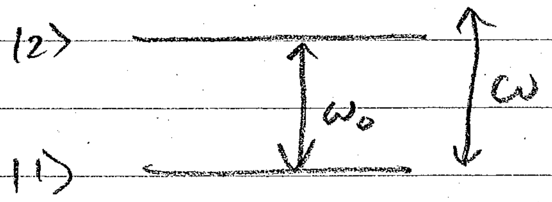


No cancellation because atoms are randomly distributed.

Result: $\bar{I}_{sc,total} = N \bar{I}_{sc}$

Resonance fluorescence.

Two-level atom.
Light tuned near the resonance



$|\omega - \omega_0|$ small

Use OBE.

$$\frac{d\tilde{\rho}_{22}}{dt} = -\frac{1}{2} i \Omega (\tilde{\rho}_{12} - \tilde{\rho}_{21}) - 2\gamma_{sp} \tilde{\rho}_{22}$$

$$\frac{d\tilde{\rho}_{12}}{dt} = \frac{1}{2} i \Omega (\tilde{\rho}_{11} - \tilde{\rho}_{22}) + \left(i(\omega_0 - \omega) - \gamma\right) \tilde{\rho}_{12}$$

Here $\gamma = \gamma_{sp} + \gamma_{coll}$

We never derived that but it's in § 2.9
and is supposed to hold also for narrow-band/single-mode

[Note: We have used γ_{coll}
so that $\tilde{\rho}$ describes a statistical average,
not a single-particle state.
Then $|\rho_{12}|^2 = |\rho_{11}||\rho_{22}|$ may be violated
& instead have $|\rho_{12}|^2 \leq |\rho_{11}||\rho_{22}|$]

Recall source-field expression

$$E^+_{sc}(\vec{r}, t) = -\frac{e \vec{D}_{12} \cdot \vec{e}_{sc}\, \omega_0^2}{4\pi \varepsilon_0 c^2 |\vec{r} - \vec{R}|} \hat{\pi}\left(t - \frac{|\vec{r} - \vec{R}|}{c}\right)$$

set $\vec{R} = 0$

take averages

$$\left\langle E_{sc}^-(\vec{r},t)\, E_{sc}^+(\vec{r},t) \right\rangle = \left(\frac{e\omega_0^2 (\vec{D}_{12}\cdot\vec{e}_{sc})}{4\pi\varepsilon_0 c^2 r} \right)^2 \rho_{22}\left(t-\frac{r}{c}\right)$$

Steady state:

$$\bar{I}_{sc} = \frac{e^2\omega_0^4 (\vec{e}_{sc}\cdot\vec{D}_{12})^2}{8\pi^2\varepsilon_0 c^3 r^2}\, \tilde{\rho}_{22}(\infty)$$

(Bar over $\bar{I}$ means cycle-averaged)

& the solution is

$$\tilde{\rho}_{22}(\infty) = \frac{\Omega^2(\gamma/4\gamma_{sp})}{(\omega-\omega_0)^2+\gamma^2+\frac{\gamma}{2\gamma_{sp}}\Omega^2}$$

1) <u>Weak incident beam</u>

$$\bar{I}_{sc} \sim \frac{\Omega^2}{(\omega-\omega_0)^2+\gamma^2}$$

but <u>note</u> that $\Omega^2 \propto \bar{I}_{in}$ [Cf. eq (4.10.11) or (2.2.10)]

so $\bar{I}_{sc} \propto \bar{I}_{in}$ weak beam

The details, and the strong-beam case, are left as a hand-in assignment!

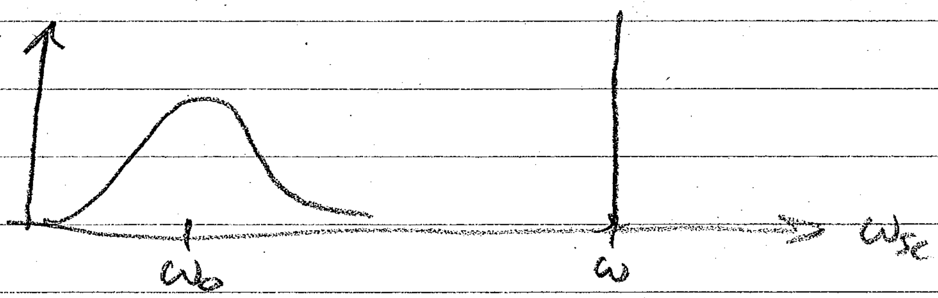
The result for the 1-order coherence is

$$g^{(1)}(\tau) = \underbrace{\frac{\gamma_{coll}}{\gamma} e^{-i\omega_0\tau-\gamma\tau}}_{\substack{\text{Chaotic part} \\ \text{Collision-broadened}}} + \underbrace{\frac{\gamma_{sp}}{\gamma} e^{-i\omega_0\tau}}_{\text{Coherent part.}}$$

Wiener-Khintchine thm: the spectrum is (15-6

$$F(\omega_{sc}) = \frac{1}{\pi} \,\mathrm{Re} \int_0^\infty d\tau \; g^{(1)}(\tau)\, e^{i\omega\tau}$$

$$= \frac{\gamma_{coll}}{\gamma} \frac{\gamma/\pi}{(\omega_0-\omega_{sc})^2+\gamma^2} + \frac{\gamma_{sp}}{\gamma}\,\delta(\omega_{sc}-\omega)$$

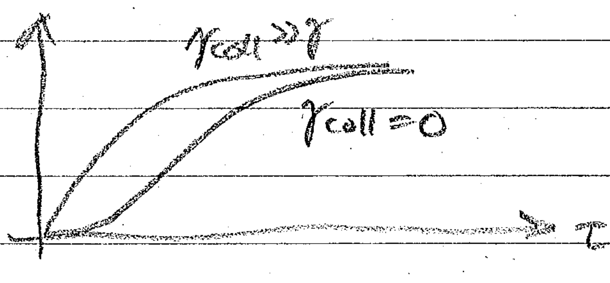


Fluorescence Rayleigh

"collisional redistribution of energy"

Scattered light is anti-bunched

Fig 8.4



Single-atom resonance fluorescence

A *single* atom: No collision or doppler broadening

$$\gamma = \gamma_{sp}$$

For simplicity, assume incident light on resonance

$$\omega = \omega_0$$

OBE

$\Rightarrow$ ... very lengthy algebra ...

$$g^{(1)}(\tau) = e^{-i\omega\tau}\left\{ \frac{2\gamma_{sp}^2}{2\gamma_{sp}^2 + \Omega^2} + \frac{1}{2}e^{-\gamma_{sp}\tau} + \frac{(\gamma_{sp} - 2i\lambda)^2 e^{-i\lambda\tau - \frac{3}{2}\gamma_{sp}\tau}}{8i\lambda(3\gamma_{sp} + 2i\lambda)} + (\lambda \to -\lambda) \right\}$$

where $\lambda = \sqrt{\Omega^2 - \frac{1}{4}\gamma_{sp}^2}$

Weak incident beam: $\lambda = i\sqrt{\frac{1}{4}\gamma_{sp}^2 - \Omega^2} \approx \frac{i}{2}\gamma_{sp}$

λ is just going to contribute to the broadening.

By Wiener-Khintchine again, get

$$F(\omega_{sc}) = \frac{\gamma_{sc}\Omega^2/\pi}{[(\omega-\omega_{sc})^2 + \gamma_{sp}^2]^2} + \text{elastic part}$$

a *squared* Lorentzian which has

$$\text{FWHM} = 2\gamma_{sp}\sqrt{\sqrt{2} - 1} \approx 2\gamma_{sp}\cdot 0.64$$

("quadrature squeezing")

2) Stronger beam $\Omega > \gamma_{sp}/2$.

Now $\lambda \in \mathbb{R}$.

Now the spectrum is

$$F(\omega_{sc}) = \frac{2\gamma_{sp}^2}{2\gamma_{sp}^2 + \Omega^2}\,\delta(\omega_{sc}-\omega) + \frac{\gamma_{sp}/2\pi}{(\omega-\omega_{sc})^2+\gamma_{sp}^2}$$

$$+ \frac{3\gamma_{sp}\lambda(\Omega^2-2\gamma_{sp}^2) + \gamma_{sp}(5\Omega^2-2\gamma_{sp}^2)(\omega+\lambda-\omega_{sc})}{8\pi\lambda(2\gamma_{sp}^2+\Omega^2)\left[(\omega+\lambda-\omega_{sc})^2+\left(\frac{3}{2}\gamma_{sp}\right)^2\right]}$$

$$+ (\lambda \to -\lambda)$$

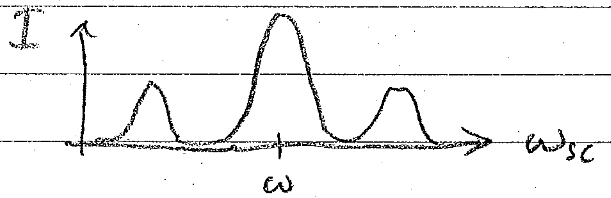
this looks complicated but for $\Omega \gg \gamma_{sp}$
then $\lambda \approx \Omega$ and

$$F(\omega_{sc}) \approx \frac{(3/8\pi)\,\gamma_{sp}}{(\omega-\Omega-\omega_{sc})^2+\left(\frac{3}{2}\gamma_{sp}\right)^2} + \frac{\frac{1}{2\pi}\gamma_{sp}}{(\omega-\omega_{sc})^2+\gamma_{sp}^2}$$

$$+ \frac{\frac{3}{8\pi}\gamma_{sp}}{(\omega+\Omega-\omega_{sc})^2+\left(\frac{3}{2}\gamma_{sp}\right)^2}$$

[Elastic part is much smaller]

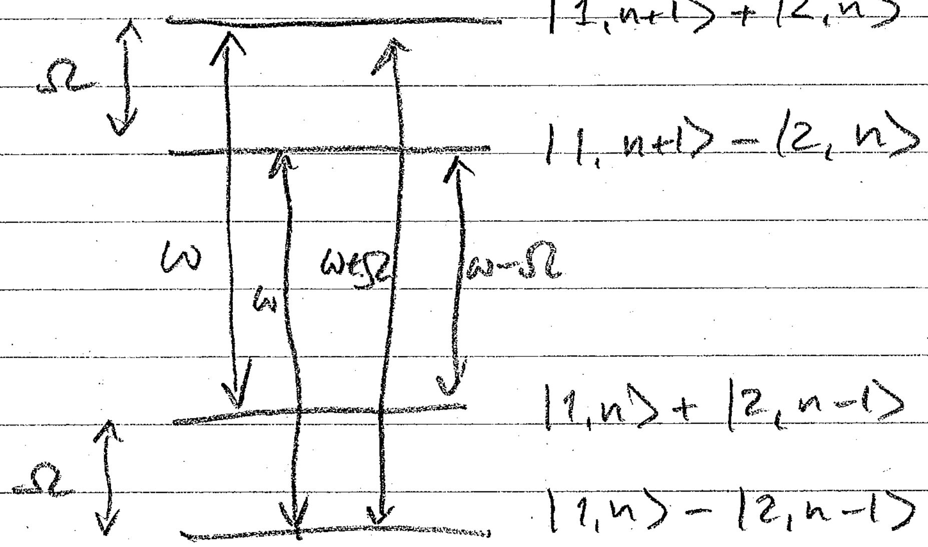
= <u>three</u> Lorentzians: at $\omega_{sc} = \omega$, $\omega_{sc} = \omega \pm \Omega$



"Mollow triplet"

Explanation: "the dynamic Stark effect"

Basically, the interaction lifts the degeneracy
$|\text{atom}, \text{radiation}\rangle$



Degenerate perturbation theory

Without H_{ED}: $|1,n\rangle \longleftrightarrow E = n\hbar\omega$
$|2,n-1\rangle \longleftrightarrow E = n\hbar\omega$

Coupling: $\langle 1,n|\hat{H}_{ED}|2,n-1\rangle = \Omega$

Then the eigenvalue problem is

$$0 = \begin{vmatrix} n\hbar\omega - \varepsilon & \Omega \\ \Omega & n\hbar\omega - \varepsilon \end{vmatrix} \Rightarrow \quad \varepsilon = n\hbar\omega \pm \Omega$$

with eigenvectors $|1,n\rangle \pm |2,n-1\rangle$

so $\hat{H}_{ED}$ lifts the degenerate eigenstates of atom + incident beam.
When this system is de-excited,
a photon is emitted in a different mode.