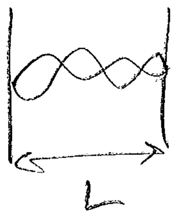
# The electromagnetic vacuum

Consider a cavity with 2 mirrors at the ends




Energy of the field, counting only modes ⊥ mirrors (1D!)

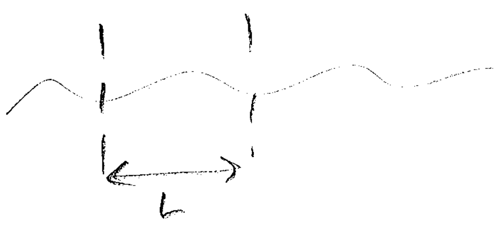
$$E_{cav} = \sum_{k\lambda} \hbar\omega_k \left(n_{k\lambda} + \frac{1}{2}\right)$$

Vacuum energy $\left[\text{noting that } \sum_{\lambda=1}^{2} = 2\right]$

$$E_{cav} = 2 \cdot \frac{1}{2} \sum_k \hbar c k = \left\{ k = \frac{n\pi}{L} \right\}$$

$$= \frac{\pi \hbar c}{L} \sum_{n=1}^{\infty} n \qquad (1D)$$

Compare this to the vacuum energy of the <u>same</u> portion of space, with <u>mirrors removed</u>.




Now any wavevector is allowed

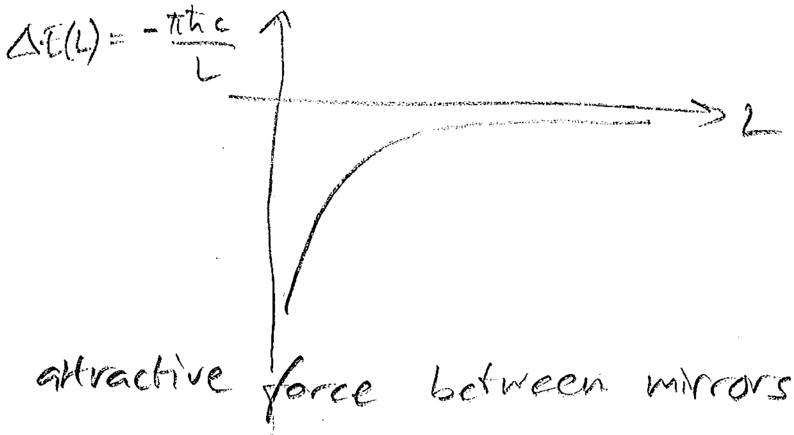
$$E_{free} = \frac{\pi \hbar c}{L} \int_0^{\infty} dn\, n$$

The energy <u>difference</u> introduced by <u>confining</u> the field is:

$$\Delta E = E_{cav} - E_{free} = \frac{\pi \hbar c}{L} \left[ \sum_{n=1}^{\infty} n - \int_0^{\infty} dn\, n \right]$$

$$= -\frac{1}{12} \frac{\pi \hbar c}{L} \qquad (1D)$$

12-2 | Energy as a function of distance between mirrors



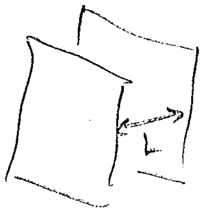

$\Rightarrow$ attractive force between mirrors!

$$\boxed{F = -\frac{\partial \Delta E}{\partial L} = -\frac{\pi \hbar c}{12 L^2} \quad (1D)}$$

the Casimir force.

Full 3D calculation:

$$\Rightarrow F = \frac{\pi^2 \hbar c}{240 L^4}$$




This is a real consequence of the quantum electromagnetic vacuum.

Has been measured experimentally (mid-1990s)!