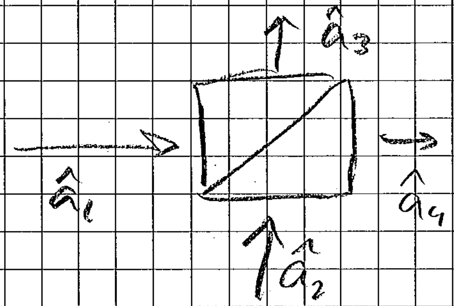
10. Quantum-mechanical beam-splitter theory



Virtue of beam splitter: create superposition states!

Reflection coeff. R
Transmission — " — T

$$\begin{cases} |R|^2 + |T|^2 = 1 \\ RT^* + TR^* = 0 \end{cases}$$

(derived in Ch. 3)

$$\hat{a}_3 = R\hat{a}_1 + T\hat{a}_2$$

$$\hat{a}_4 = T\hat{a}_1 + R\hat{a}_2$$

Reverse:

$$\hat{a}_1 = R^*\hat{a}_3 + T^*\hat{a}_4$$

$$\hat{a}_2 = T^*\hat{a}_3 + R^*\hat{a}_4$$

Independent input fields: $[\hat{a}_1, \hat{a}_2^\dagger] = 0$

The electric fields

$$\hat{E}_3(\chi) = |R|\hat{E}_1(\chi - \phi_R) + |T|\hat{E}_2(\chi - \phi_T)$$

$$\hat{E}_4(\chi) = |T|\hat{E}_1(\chi - \phi_T) + |R|\hat{E}_2(\chi - \phi_R)$$

where ϕ_R and ϕ_T are the phases of the complex numbers R and T

10-2 Photon numbers

$$\hat{n}_3 = |R|^2 \hat{n}_1 + R^* T \hat{a}_1^\dagger \hat{a}_2 + R T^* \hat{a}_2^\dagger \hat{a}_1 + |T|^2 \hat{n}_2$$

$$\hat{n}_4 = |T|^2 \hat{n}_1 + R T^* \hat{a}_1^\dagger \hat{a}_2 + R^* T \hat{a}_2^\dagger \hat{a}_1 + |R|^2 \hat{n}_2$$

& variances (assuming arm 2 in vacuum state, for simplicity)

$$(\Delta n_3)^2 = |R|^4 (\Delta n_1)^2 + |R|^2 |T|^2 \langle n_1 \rangle$$

$$(\Delta n_4)^2 = |T|^4 (\Delta n_1)^2 + |T|^2 |R|^2 \langle n_1 \rangle$$

- Single-photon input:

One photon in arm 1, vacuum in arm 2:

$$|1\rangle_1 |0\rangle_2 = \hat{a}_1^\dagger |0\rangle$$

But $\hat{a}_1^\dagger = R \hat{a}_3^\dagger + T \hat{a}_4^\dagger$

$$|1\rangle_1 |0\rangle_2 = R |1\rangle_3 |0\rangle_4 + T |0\rangle_3 |1\rangle_4$$

The BS creates an entangled state.

(a very important concept in quantum information)

Recall quantum measurement postulate:

A measurement collapses the state onto the corresponding eigenstate

If we look in arm 3 and see no photons, then

$$|\text{final}\rangle = N \cdot {}_3\langle 0| \left\{ R |1\rangle_3 |0\rangle_4 + T |0\rangle_3 |1\rangle_4 \right\} = |1\rangle_4$$

<u>Expectation values of photon number</u>

corresponds to average over many measurements:

$$\langle n_3 \rangle = \langle 0_2 | \langle 1_1 | \hat{n}_3 | 1_1 \rangle | 0_2 \rangle$$

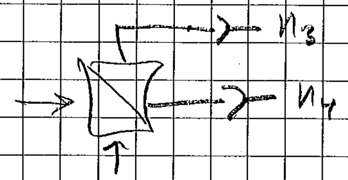
$$= \langle 0_2 | \langle 1_1 | \left(R^* \hat{a}_1^\dagger + T^* \hat{a}_2^\dagger \right) \left(R \hat{a}_1 + T \hat{a}_2 \right) | 1_1 \rangle | 0_2 \rangle$$

$$= |R|^2$$

$$\langle n_4 \rangle = |T|^2$$

that was not so hard to guess, was it?

- <u>Hanbury Brown - Twiss experiment</u>



measures $\langle n_3 n_4 \rangle$

For one-photon input, get

$$\langle n_3 n_4 \rangle = \ldots = 0$$

... could never happen in classical theory!

Recall <u>classical</u>:

$$\langle I_3 \rangle = |R|^2$$

$$\langle I_4 \rangle = |T|^2$$

$$\langle I_3(t) I_4(t+\tau) \rangle = \langle I_3 \rangle \langle I_4 \rangle g^{(2)}(\tau)$$

$$\langle I_3(t) I_4(t) \rangle = \langle I_3 \rangle \langle I_4 \rangle \underbrace{g^{(2)}(0)}$$

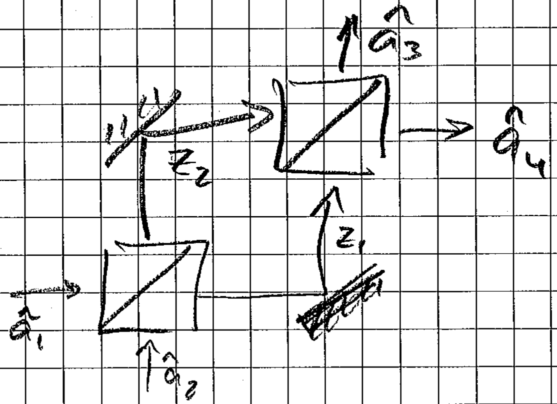
$= 1$ for stable wave

$= 2$ for chaotic light

$\Longrightarrow$ arbitrary input.

10-4

Mach-Zehnder interferometer



$$\hat{a}_3 = R_{MZ}\, \hat{a}_1 + T_{MZ}\, \hat{a}_2 \qquad \hat{a}_4 = T_{MZ}\, \hat{a}_1 + R'_{MZ}\, \hat{a}_2$$

$$\begin{cases} R_{MZ} = R^2 e^{ikz_1} + T^2 e^{ikz_2} & R'_{MZ} = T^2 e^{ikz_1} + R^2 e^{ikz_2} \\ T_{MZ} = RT\left(e^{ikz_1} + e^{ikz_2}\right) & \end{cases}$$

Calculate mean photon number in arm 4

$$\langle n_4 \rangle = |T_{MZ}|^2 = |R|^2 |T|^2 \cdot 4\cos^2\left[k(z_1 - z_2)/2\right]$$

= <u>same</u> as classical! <u>interference fringes.</u>

- For arbitrary input $|\psi_1\rangle$ in arm 1, obtain

$$\langle n_4 \rangle = |R|^2 |T|^2 \cdot \langle n \rangle\, 4\cos^2\left[k(z_1 - z_2)/2\right]$$

$\Rightarrow$ it does not matter if we have many photons simultaneously, or one at a time, several times.

$\therefore$ <u>interference is a single-photon effect!</u>

Each single photon excites both paths of the MZ. It does <u>not</u> go either way.

- Now, let arm 1 be in an arbitrary state & arm 2 still in vacuum $|\psi_1\rangle|0_2\rangle$

We get for single BS:

$$\langle n_3\rangle = \langle 0_2|\langle\psi_1|\left(|R|^2 a_1^\dagger a_1\right)|\psi_1\rangle|0_2\rangle$$

$$= |R|^2\langle n_1\rangle$$

$$\langle n_4\rangle = |T|^2\langle n_1\rangle$$

HBT experiment:

$$\langle n_3 n_4\rangle = \cdots = |R|^2|T|^2\langle n_1(n_1-1)\rangle$$

We can define a correlation function

$$g_{ab}^{(2)}(\tau) = \frac{\langle \hat{a}_a^\dagger \hat{a}_b^\dagger \hat{a}_b \hat{a}_a\rangle}{\langle \hat{a}_a^\dagger \hat{a}_a\rangle\langle \hat{a}_b^\dagger \hat{a}_b\rangle}$$

{ Remember this was time-independent in single-mode case! }

Then

$$g_{34}^{(2)} = \frac{\langle n_3 n_4\rangle}{\langle n_3\rangle\langle n_4\rangle}$$

and

$$g_{11}^{(2)} = \frac{\langle a_1^\dagger a_1^\dagger a_1 a_1\rangle}{\langle n_1\rangle^2} \overset{\text{use } [a_1, a_1^\dagger]=1}{=} \frac{\langle n_1(n_1-1)\rangle}{\langle n_1\rangle^2}$$

Observe

$$g_{34}^{(2)}(\tau) = \frac{\langle n_3 n_4\rangle}{\langle n_3\rangle\langle n_4\rangle} = \frac{|R|^2|T|^2\langle n_1(n_1-1)\rangle}{|R|^2\langle n_1\rangle\,|T|^2\langle n_1\rangle} = g_{11}^{(2)}(\tau)!$$

= classical result.

10.6 Coherent-state input.

Let $|\psi_1\rangle = \hat{D}_1(\alpha)|0\rangle = e^{\alpha \hat{a}_1^\dagger + \alpha^* \hat{a}_1}|0\rangle$

$= \exp\{\alpha(R\hat{a}_3^\dagger + T\hat{a}_4^\dagger) + \alpha^*(R^*\hat{a}_3 + T^*\hat{a}_4)\}|0\rangle$

$= \hat{D}_3(R\alpha)\hat{D}_4(T\alpha)|0\rangle$

Observe! $e^{\alpha R\hat{a}_3^\dagger + \alpha^* R^* \hat{a}_3 + \alpha T\hat{a}_4^\dagger + \alpha^* T^* \hat{a}_4}$

$= e^{\alpha R\hat{a}_3^\dagger + \alpha^* R^* \hat{a}_3} \cdot e^{\alpha T\hat{a}_4^\dagger + \alpha^* T^* \hat{a}_4}$

We could factorize only because 3 & 4 are independent: $[\hat{a}_3, \hat{a}_4^\dagger] = 0$

$|\psi_1\rangle = |R\alpha_3\rangle|T\alpha_4\rangle$

then

$\langle n_3 n_4\rangle = |R\alpha|^2|T\alpha|^2 = \langle n_3\rangle\langle n_4\rangle$

(but $\langle n_3 n_4\rangle = |R|^2|T|^2\langle n_1(n_1-1)\rangle$

$= |R|^2|T|^2(\langle n_1^2\rangle - \langle n_1\rangle)$ $\left\{\langle n_1^2\rangle = \langle n_1\rangle^2 - \langle n_1\rangle \text{ for coh. states!}\right\}$

$= |R|^2|T|^2\langle n_1\rangle^2$)

∴ Same result as 2 independent sources.

= same as coherent classical light.

Nonclassical light

Poissonian distribution: $\langle n^2 \rangle = \langle n \rangle$

A prob. distribution is called super-Poissonian if $\langle n^2 \rangle \geq \langle n \rangle$

and sub-Poissonian if $\langle n^2 \rangle \leq \langle n \rangle$

Classical light is always super-P.

$$1 \leq g^{(2)}(0) \leq \infty \quad \text{(classical)}$$

Quantum:

$$1 - \frac{1}{\langle n \rangle} \leq g^{(2)}(0) < \infty \quad \text{for } \langle n \rangle \geq 1$$

$$0 \leq g^{(2)}(0) \quad \text{for } \langle n \rangle < 1 \text{ \& single-mode.}$$

[Disregard Eq.(5.10.4), it's just confusing]

Classically: $\langle I^2 \rangle$

May measure I & leave it undisturbed

→ measure it again

Quantum: $\langle n(n-1) \rangle$

First measurement absorbs a photon

*... so q.m. light can be sub-Poissonian

→ e.g. in photon number states.