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June 3, 2010

Tentamen i Växelverkan mellan Ljus och materia
Quantum Optics

Date: 2010-06-03, 09.00-15.00, Sal 4

Teacher: Emil Lundh

5 problems on 2 pages. Maximum grade for each problem is 1. For maximum grade the calculations and the reasoning must be fully accounted for in a way that is easy to follow.

Allowed aids: Calculator, Physics Handbook (Nordling/Österman), Beta Mathematics Handbook, Taschenbuch der Mathematik (Bronstein).

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1. In this problem you are asked to derive the semiclassical optical Bloch equations in the same way as was done in the book and the lectures. Assume a two-level atom and write its general state as

$$|\Psi(t)\rangle = C_1(t)e^{-iE_1t/\hbar}|1\rangle + C_2(t)e^{-iE_2t/\hbar}|2\rangle.$$

Assume that there is radiation at frequency ω. With electric dipole coupling between atom and field, the equations of motion for the coefficients are

$$i\frac{dC_1}{dt} = \Omega\cos(\omega t)e^{-i\omega_0 t}C_2,$$
$$i\frac{dC_2}{dt} = \Omega\cos(\omega t)e^{i\omega_0 t}C_1,$$

where $\omega_0 = (E_2 - E_1)/\hbar$ and Ω is the Rabi frequency.
From this point, define the density matrix and derive the equation of motion for one of its elements (any one of your choice). Invoke the rotating-wave approximation. I only want you to derive the equation of motion for *one* of the elements, because the other ones are similar and the calculation will become tedious.

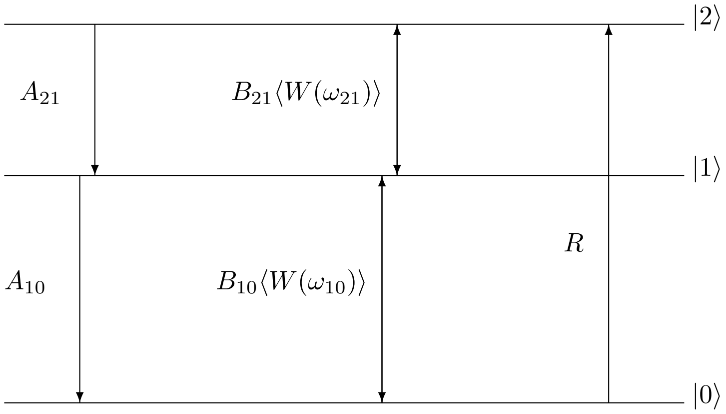
2. A coherent state in a single mode can be defined as

$$|\alpha\rangle = e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle,$$

where $|n\rangle$ is a number state.
(a) Calculate the expectation value and the standard deviation of the number of photons in the coherent state.
(b) Can they make a coherent state, or at least something close to a coherent state, for visible light in experiment? Explain how the real experimental states differ from the ideal coherent state above.

3. State the quantum measurement postulate. Do it by giving expressions for a general quantum state before and after measurement, $|\Psi(t^-)\rangle$ and $|\Psi(t^+)\rangle$, respectively.

4. The (mathematically) simplest way to construct a laser is by using a three-level atom. The levels are called $|0\rangle$, $|1\rangle$, and $|2\rangle$, where $|0\rangle$ has the lowest energy and $|2\rangle$ has the highest energy. Levels $|1\rangle$ and $|2\rangle$ are coupled via stimulated and spontaneous emission and absorption, and so are levels $|0\rangle$ and $|1\rangle$. The transition between $|0\rangle$ and $|2\rangle$ is dipole forbidden, but atoms are "pumped" from $|0\rangle$ to $|2\rangle$ at a rate R by some other means (like electron bombardment). The rates of stimulated emission and absorption are called $B_{ij}\langle W(\omega_{ij})\rangle$ and A_{ij}, respectively.



(a) If we are to construct a laser, which of those rates need to be much smaller or much larger than the others? Which atomic population should be the biggest and which should be the smallest?
(b) Write down a rate equation for the population of atoms in state 2. If some quantity is assumed small (according to your answer above), you can put it to zero in the rate equation.

5. Explain what bunched and anti-bunched light is. Explain what Poissonian, sub- and super-Poissonian light is. Draw sketches if you think it helps. Draw a special sketch of the degree of second-order coherence of a wave that is super-Poissonian and anti-bunched.