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EXAM

Växelverkan mellan ljus och materia — Quantum Optics, 5p
2006–08–25, at 9.00–15.00
Östra paviljongen, room 5 **(bokningskod: 86172)**

Allowed aids:

- Physics Handbook (Nordling/Österman)
- Beta - Mathematical handbook
- Pocket calculator
- Three A4 pages of handwritten notes (with text on both sides, but not including solved problems and examples).

Every problem will give a maximun of 1.00 points. The calculations and the reasoning must always be fully accounted for in a way that is easy to follow. Write your name on *all* submitted papers.

Good luck!

1 A coherent state is defined by:

$$|\alpha\rangle = \exp\left(-\frac{1}{2}|\alpha|^2\right)\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle \, . \tag{1}$$

What is the expectation value of the number of photons for this state, and what is the quantum mechanical uncertainty? The answer must be well (mathematically) motivated.

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2 Describe qualitatively the broadening mechanisms, 'natural linewidth', 'Doppler broadening', 'collisional broadening' and 'power broadening'. What different lineshapes do these lead to, respectively? Under which respective circumstances are they important? For each of them give an example where this particular mechanism will be the most important broadening mechanism of a spectral line.

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3 A light source consists of a large number of independent atoms, all emitting light with the frequency ω_0. The atoms all move with the same speed, v_0, but in a random direction. Calculate:

a. the degree of first-order coherence, $g^{(1)}(\tau)$;

b. and the degree of second-orde coherence, $g^{(2)}(\tau)$.

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4 A photon pair state is created that excites two different continuous-mode fields. These two photons are arranged such that they excite the two different input arms in a beam splitter. Thus, the input state of the beam splitter can be written as

$$|(1_1, 1_2)_\beta\rangle = \int\int \beta(t,t')\hat{a}_1^\dagger(t)\hat{a}_2^\dagger(t')\mathrm{d}t\mathrm{d}t' \quad . \tag{2}$$

The reflectance of the beam splitter is $|\mathcal{R}|^2$ and the transmittance is $|\mathcal{T}|^2$. $\mathcal{R}$ and $\mathcal{T}$ are respectively the complex reflection and transmission coefficients.

5a) Rewrite eq. (2) in a form where it contains the creation operators for the output arms instead of those for the input arms.

5b) It is a 50:50 beam splitter, which means that the reflectance and the transmittance are equal ($|\mathcal{R}|^2 = |\mathcal{T}|^2 = 1/2$). Moreover, the two states overlap perfectly in time. In other words, the joint overlap integral is unity:

$$|J|^2 = \int\int \beta^*(t,t')\beta(t',t)\mathrm{d}t\mathrm{d}t' = 1 \quad . \tag{3}$$

For this particular case, the probability is for finding one photon in each of the two output arms at a measurement is zero. Interpret this result. What does it mean, and why does this happen?

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5 A three-level system, as in figure 1 is used in order realize a laser.

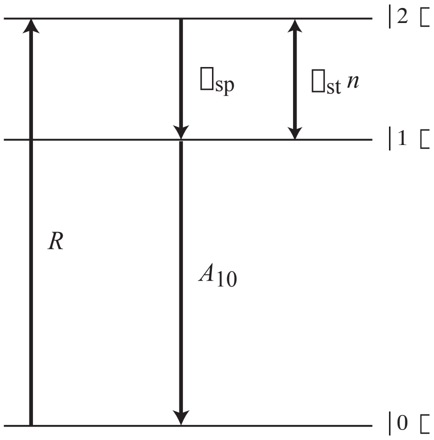

Figure 1: Atomic energy-level scheme for a three-level laser showing the relevant transition rates.

The rates indicated in the figure are; the rate of pumping from $|0\rangle$ to $|2\rangle$, R ($|0\rangle \leftrightarrow |2\rangle$ is dipole forbidden, so this pumping is done with for example electron bombardment); the spontaneous decay from $|1\rangle$ to $|0\rangle$, A_{10}; the spontaneous emission rate from $|2\rangle$ to $|1\rangle$, Γ_{sp}; and the rates for absorption and stimulated emission between states $|1\rangle$ and $|2\rangle$, $\Gamma_{\text{st}}n$. Another important rate is the loss of cavity photons due to imperfect reflections in the output mirrors, Γ_{cav}. Of all these rates, A_{10} is by far greater than all the others.

a. Draw an energy-level diagram for the the lasing mode, involving numbers of photons. Indicate by arrows and symbols the relevant transitions that contribute to the probability of having n photons in the lasing mode.

b. Write down rate equations for the probability of having n photons in the lasing mode, and for the population in state $|2\rangle$.

c. What is the population of state $|2\rangle$ at steady-state? How does it depend on n, and how can that be interpreted physically?