Ass 5

$$\bar{I}_{sc} = \frac{e^2\omega_0^4(\vec{e}_{sc}\cdot\vec{D}_{12})^2}{8\pi^2\varepsilon_0 c^3 r^2}\rho_{22}(\infty)$$ scattering into surface element.

Rate per surface element $r^2 d\Omega_{sc}$

$$\frac{dR}{d\Omega_{sc}} = \sum_\lambda \frac{I_{sc}}{\hbar\omega_0} = \frac{2I_{sc}}{\hbar\omega_0}$$ (summed over polarizations)

total rate

$$R = 2\int \frac{I_{sc}}{\hbar\omega_0} r^2 d\Omega_{sc} = \frac{2e^2\omega_0^3}{8\pi^2\varepsilon_0 c^3\hbar}\rho_{22}(\infty)\underbrace{\int(e_{sc}\cdot D_{12})^2\, d\Omega_{sc}}_{=4\pi/3\, D_{12}^2}$$

$$= \underbrace{\frac{e^2\omega_0^3 D_{12}^2}{3\pi\varepsilon_0 c^3\hbar}}\rho_{22}(\infty)$$

Note! $= 2\gamma_{sp}\rho_{22}(\infty)$

[Failure to sum over polarizations -0.1 p]

$\rho_{22}(\infty)$ needs to be averaged over all incoming atoms

If you assumed polarized atoms (all $\vec{D}_{12}$ equal), it's ok

because it was not given in the text.

However, failure to even discuss polarization of atoms

gives -0.2 p.

For a large number of randomly polarized atoms:

$$\langle\rho_{22}(\infty)\rangle = N\int d\Omega_{in}\frac{(\gamma/4\gamma_{sp})\Omega^2}{(\omega_0-\omega)^2+\gamma^2+(\gamma/2\gamma_{sp})\Omega^2}$$

where $\Omega^2 = \frac{2e^2}{\varepsilon_0 c\hbar^2}(\vec{e}_i\cdot\vec{D}_{12})\bar{I}_{in}$

$\langle\rho_{22}(\infty)\rangle$ gives a tanh function in general

<u>Weak</u> beam $\Omega \ll \gamma$

$$\langle \rho_{22}(\infty) \rangle = N \int d\Omega_{in} \frac{(\gamma/4\gamma_{sp})}{(\omega_0-\omega)^2+\gamma^2} \cdot \frac{2e^2}{\varepsilon_0 c \hbar^2} \overline{I}_{in} \underbrace{(\vec{e}_i \cdot \vec{D}_{12})^2}_{4\pi/3}$$

$$= N \frac{(\gamma/4\gamma_{sp})}{(\omega_0-\omega)^2+\gamma^2} \underbrace{\frac{2e^2}{\varepsilon_0 c \hbar^2} \frac{4\pi}{3} D_{12}^2}_{\frac{8\pi^2 c^2}{\hbar \omega_0^3} \gamma_{sp}} \overline{I}_{in}$$

$$= N \frac{\gamma}{(\omega_0-\omega)^2+\gamma^2} \frac{2\pi^2 c^2}{\hbar \omega_0^3} \overline{I}_{in}$$

$$R = N \cdot \frac{2\gamma\gamma_{sp}}{(\omega_0-\omega)^2+\gamma^2} \cdot \quad \text{— " —}$$

<u>Strong</u> beam $\Omega \gg \gamma$

$\langle \rho_{22}(\infty) \rangle = N \cdot \frac{1}{2}$ no dependence on angle

$\Rightarrow R = \gamma_{sp} N$

— o —

<u>Note:</u> Taking $\vec{e}_1 \perp \vec{D}_{12}$ and then $\vec{e}_2 \perp \vec{e}_1$ and integrating over angles – works in principle. But now $\vec{e}_2$ is a complicated function of angle and the result is <u>not</u> just $\frac{4\pi}{3}$!

<u>Better</u> to take $\vec{e}_1, \vec{e}_2$ free & take

$$2 \cdot \int d\Omega_1 (\vec{e}_1 \cdot \vec{D}_{12}) = 2 \frac{4\pi}{3}.$$

(If one wants to, one may take $\vec{e}_1 = \frac{\vec{r} \times \vec{k}_{in}}{|\vec{r}||\vec{k}_{in}|}$, $\vec{e}_2 = \frac{\vec{r} \times \vec{e}_1}{|\vec{r}|}$

and $\vec{D}_{12}$ to be a general vector – it works and you eventually get $\frac{8\pi}{3} |\vec{D}_{12}|$.)