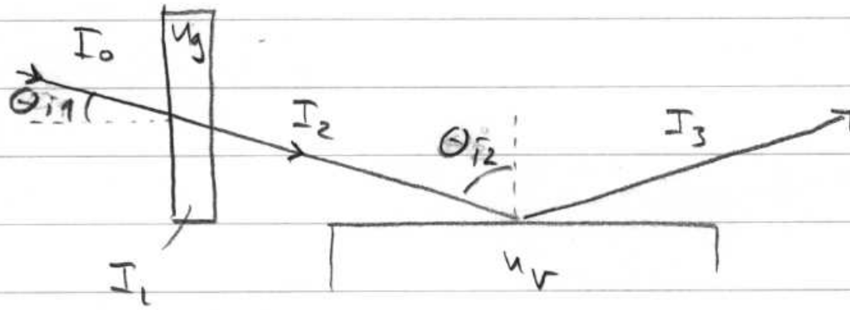
Preliminära lösningar

2003-03-19

Vågfysik och Optik

(1) vi antar att det bara är ett glas!!



$\theta_{i1} = 10°$
$\theta_{i2} = 80°$
$n_g \approx 1.5$
$n_v \approx 1.33$

Opolariserat ljus : $I_{0\parallel} = I_{0\perp} = I_0/2$

$I_1 = I_0 (1 - R_{01})$

ytan plan + symmetri => $R_{12} = R_{01}$

=> $I_2 = I_0 (1 - R_{01})^2$

$I_3 = I_0 \cdot R_{23} \cdot (1 - R_{01})^2$

För alla ytor är infallsplanet detsamma
=> samma komp.-uppdelning ($\parallel$ och $\perp$) gäller överallt

Vinkeln inne i Fönsterrutan : θ_{t1}
Vinkeln under vattnet : θ_{t2}

Snell => $1 \cdot \sin \theta_{i1} = n_g \cdot \sin \theta_{t1}$
=> $\theta_{t1} = 6.65°$

$1 \cdot \sin \theta_{i2} = n_v \cdot \sin \theta_{t2}$
=> $\theta_{t2} = 47.8°$

$$R_{01-\parallel} = \left(\frac{\tan(\theta_{i1} - \theta_{t1})}{\tan(\theta_{i1} + \theta_{t2})} \right)^2 = 0.0384$$

$$R_{23-\parallel} = \left(\frac{\tan(\theta_{i2} - \theta_{t2})}{\tan(\theta_{i2} + \theta_{t2})} \right)^2 = 0{,}239$$

$$\Rightarrow I_{3-\parallel} = \frac{I_0}{2} \cdot 0{,}239 \cdot (1 - 0.0384)^2 = 0{,}11\, I_0$$

$$R_{01-\perp} = \left(\frac{\sin(\theta_{i1} - \theta_{t1})}{\sin(\theta_{i1} + \theta_{t1})} \right)^2 = 0{,}0417$$

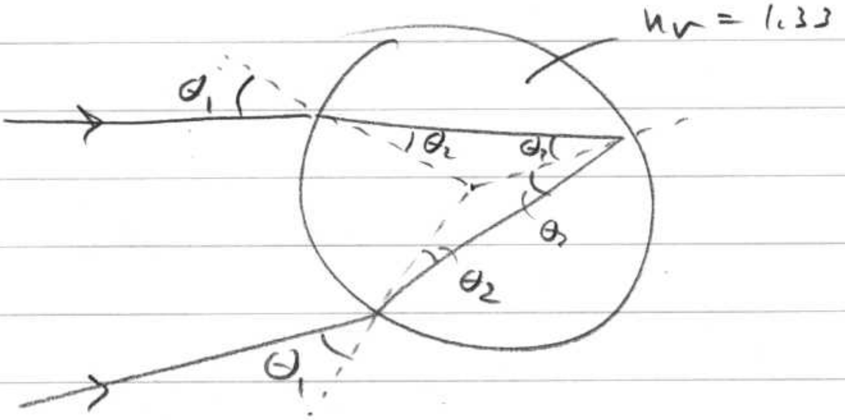
$$R_{23-\perp} = \left(\frac{\sin(\theta_{i2} - \theta_{t2})}{\sin(\theta_{i2} + \theta_{t2})} \right)^2 = 0{,}455$$

$$\Rightarrow I_{3-\perp} = 0{,}21\, I_0$$

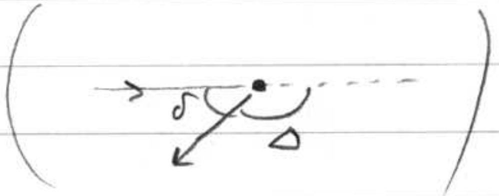
Total intensitet: $I_{tot-3} = I_{3\perp} + I_{3\parallel} = 0{,}32\, I_0$

Ljuset är delvis polariserat. Den största komponenten är $\perp$-mot infallsplanet, dvs horisontellt. Den komp. är $\frac{0{,}21}{0{,}11} = 1{,}9$ gånger större än den andra

(2) Regnbuen oppkomme då retlelstrålen i en vattdråpes bakkant har en maximal vinkel.



Total avbøyning: Δ



$$\Delta = (\theta_1 - \theta_2) + (180° - 2\theta_2) + (\theta_1 - \theta_2) =$$
$$= 180° + 2\theta_1 - 4\theta_2$$

$$\delta = 180° - \Delta = 4\theta_2 - 2\theta_1$$

Snell $\Rightarrow$ $$1 \cdot \sin\theta_1 = n_v \cdot \sin\theta_2$$
$$\Rightarrow \theta_2 = \arcsin\left(\frac{\sin\theta_1}{n_v}\right)$$

$$\Rightarrow \delta(\theta_1) = 4 \arcsin\left(\frac{\sin\theta_1}{n_v}\right) - 2\theta_1$$

Maximum for $\frac{d}{d\theta_1}\left(\delta(\theta_1)\right) = 0$

$$\frac{d\delta}{d\theta_1} = 4 \cdot \frac{1}{\sqrt{1 - \left(\frac{\sin\theta_1}{n_v}\right)^2}} \cdot \frac{\cos\theta_1}{n_v} - 2 = 0$$

$$\Rightarrow 4\,\frac{\cos\theta_1}{n_r} = 2\sqrt{1-\left(\frac{\sin\theta_1}{n_r}\right)^2}$$

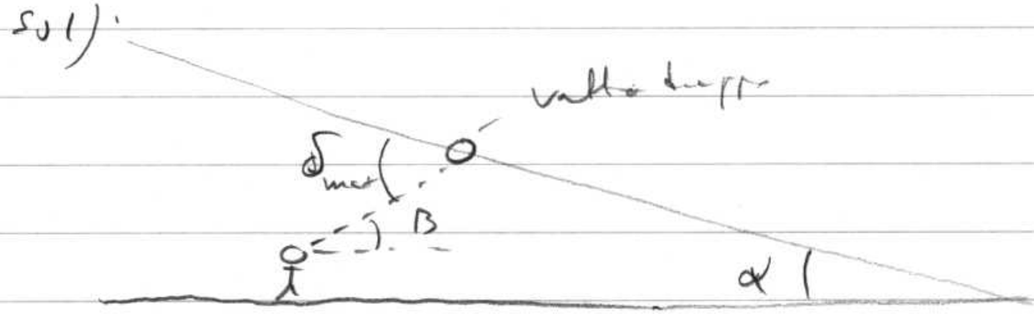
$$\frac{4}{n_r^2}\cos^2\theta_1 = 1 - \frac{1}{n_r^2}\sin^2\theta_1$$

$$n_r^2 = 4\cos^2\theta + \sin^2\theta = 4\cos^2\theta_1 + \left(1-\cos^2\theta_1\right)$$

$$n_r^2 = 3\cos^2\theta_1 + 1$$

$$\Rightarrow \theta_1 = \arccos\left(\frac{1}{3}(n_r^2-1)\right)^{1/2} = 59{,}6^\circ$$ som extremum

$$\Rightarrow \delta_{max} = \delta(59{,}6) = 42{,}5^\circ$$



Solhöjd : α

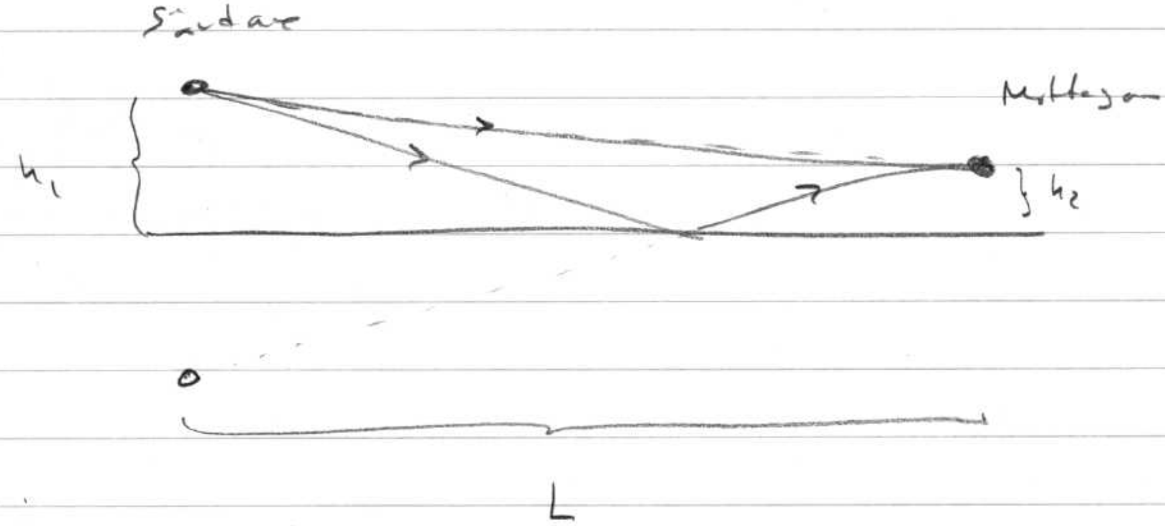
vinkeln man tittar på regnbågen : β

Geometri $\Rightarrow$ $\alpha + \beta + (180^\circ - \delta_{max}) = 180^\circ$

$$\Rightarrow \delta_{max} = \alpha + \beta$$

$$\Rightarrow \beta = \delta_{max} - \alpha =$$

3



$h_1 = \cancel{10000}\ 9000$ m, $h_2 = 100$ m, $L = \cancel{80}\ 60$ km

Direktstrålens väg: r_1

Reflekterade strålens väg: r_2

$$r_1^2 = L^2 + (h_1 - h_2)^2, \Rightarrow r_1 = 80\,610 \text{ m}$$

$$r_2^2 = L^2 + (h_1 + h_2)^2, \Rightarrow r_2 = 80\,635 \text{ m}$$

Vägskillnad: $d = r_2 - r_1 = 24.8$ m

Fasskillnad: $\varphi = \frac{2\pi}{\lambda} \cdot d + \pi$

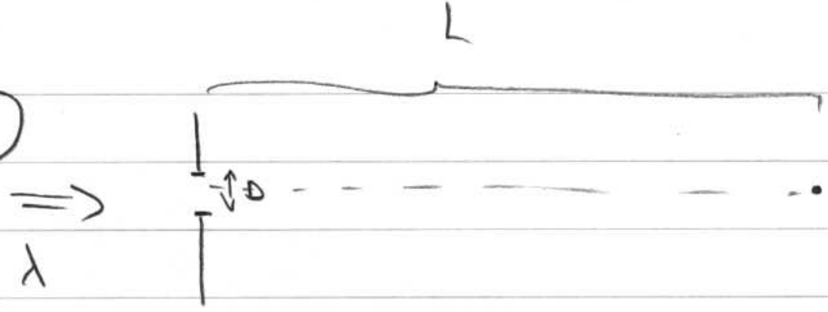
Destruktiv interferens: $\varphi = (2m+1)\pi$, m = heltal

$$\Rightarrow \frac{2\pi}{\lambda} d + \pi = 2\pi m + \pi$$

$$d = m\lambda = m \cdot \frac{c}{\nu} \quad \Rightarrow \quad \nu = \frac{mc}{d}$$

Lägsta frekvens $(m=1) \Rightarrow \nu_{min} \approx 12$ MHz

4



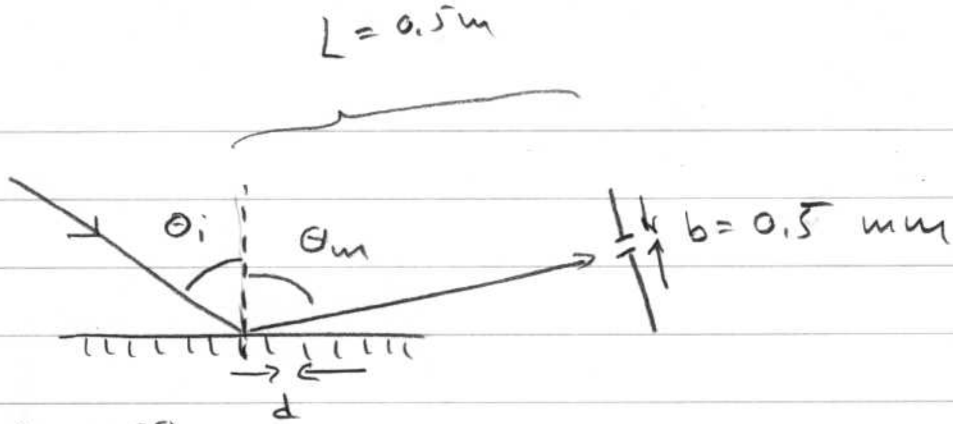
$L =$ $\qquad\qquad$ $\lambda =$

En mörk punkt uppkommer till exempel vid precis två Fresnelzoner syns i öppningen.

$$\left(\frac{D}{2}\right)^2 = m\lambda L \quad , \quad m = 2$$

$$D = 2\sqrt{2\lambda L} = 2.23 \text{ mm}$$

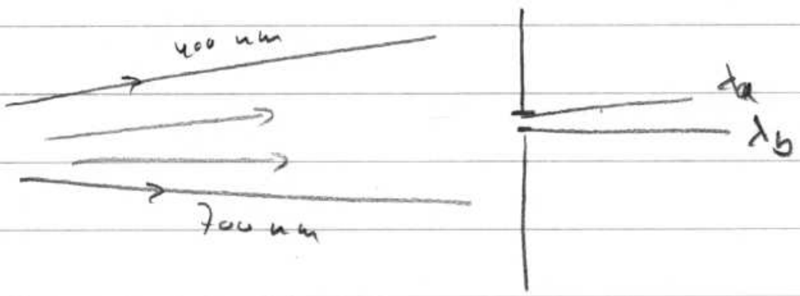
5



$\theta_i = 45^\circ$

300 ritsar per mm $\Rightarrow d = \frac{1\,mm}{300} = 3{,}33\,\mu m$

Gitterekv: $d(\sin\theta_m - \sin\theta_i) = m\lambda = 1\lambda$



bandbredd på ljuset som kommer igenom spalten: $\Delta\lambda = \lambda_a - \lambda_b$

$$\begin{cases} \sin\theta_a = \frac{\lambda_a}{d} + \sin\theta_i \\ \sin\theta_b = \frac{\lambda_b}{d} + \sin\theta_i \end{cases}$$

$$\frac{1}{d}(\lambda_a - \lambda_b) = \sin\theta_a - \sin\theta_b =$$

$$= 2\cos\frac{\theta_a+\theta_b}{2}\sin\frac{\theta_a-\theta_b}{2}$$

$\theta_a - \theta_b$ litet $\Rightarrow \sin\frac{\theta_a-\theta_b}{2} \approx \frac{\theta_a-\theta_b}{2} \approx \frac{1}{2}\cdot\frac{b}{L}$

$$\frac{\theta_a+\theta_b}{2} \approx \theta_{mitt}$$

där θ_{mitt} svarar mot en våglängd mitt i våglängdsområdet

$$\sin \theta_{mitt} = \frac{\lambda_{mitt}}{d} + \sin \theta_i \quad , \lambda_{mitt} \approx 550 \text{ nm}$$

$$\Rightarrow \theta_{mitt} \approx 60{,}7^\circ$$

$$\lambda_a - \lambda_b = d \cdot 2 \cos \theta_{mitt} \cdot \frac{1}{2} \frac{b}{L} =$$

$$= 1{,}6 \text{ nm}$$

$$\ell = \frac{\lambda^2}{\Delta \lambda} \approx \frac{\lambda_{mitt}^2}{\lambda_a - \lambda_b} = 0{,}19 \text{ mm}$$

6

lins 1

$R_{1a} = R_{1b} = -0,7\,m$

$n = 1,52$

$$\frac{1}{f_1} = (n-1)\left(\frac{1}{-R_{1a}} + \frac{1}{-R_{1b}}\right) \Rightarrow f_1 = -0,67\,m$$

lins 2

$R_{2a} = \cancel{0,5}\ 4\ m$

$R_{2b} = \infty$

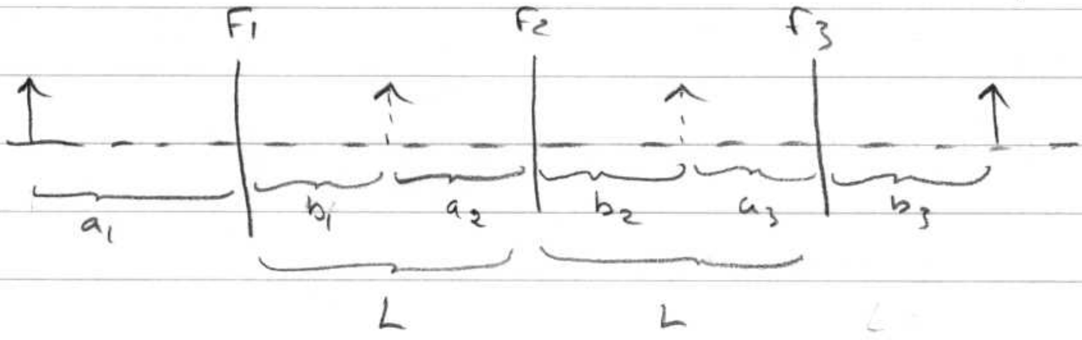
$$\frac{1}{f_2} = (n-1)\left(\frac{1}{R_{2a}} + \frac{1}{R_{2b}}\right) \Rightarrow f_2 = \cancel{0,96}\ 0.769\,m$$

lins 3

$R_{3a} = 0,5\,m$

$R_{3b} = -1,0\,m$

$$\frac{1}{f_3} = (n-1)\left(\frac{1}{R_{3a}} + \frac{1}{R_{3b}}\right) \Rightarrow f_3 = \cancel{0,52}\ 1,92\,m$$



$L = 0.4\,m$, $b_3 = 0,15\,m$, $a_1 = ?$

Gauss linsformel: $\frac{1}{f} = \frac{1}{a} + \frac{1}{b}$

3:e linsen: $\frac{1}{a_3} = \frac{1}{f_3} - \frac{1}{b_3} \Rightarrow a_3 = -\cancel{4,7}\,m$ $-0.163\,m$

$b_2 = L - a_3 = \cancel{-4,3}\,m$ $0,563\,m$

2:a: $\frac{1}{a_2} = \frac{1}{f_2} - \frac{1}{b_2} \Rightarrow a_2 = \cancel{0,79}\,m$ $-2,096\,m$

$2{,}496$ m

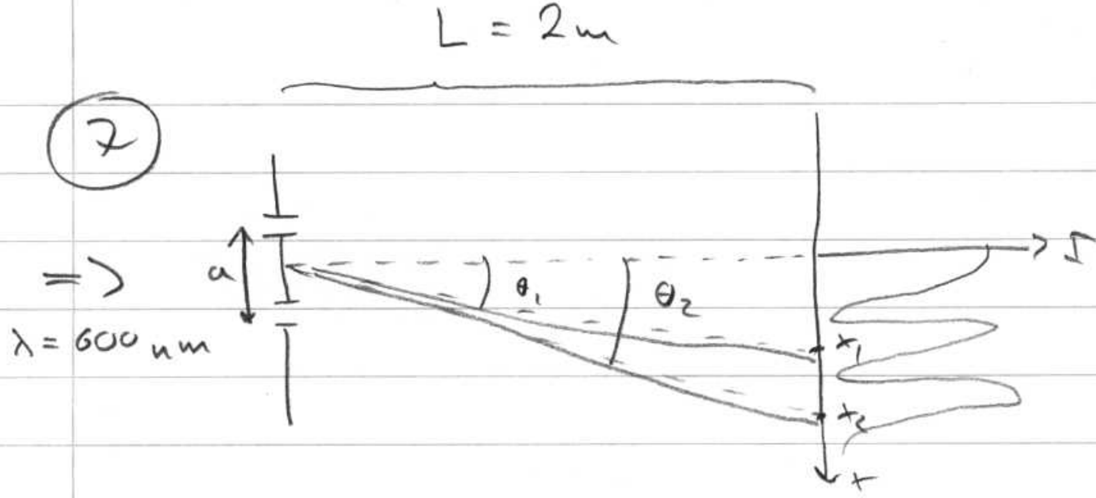
$b_1 = L - a_2 = \cancel{-0{,}39}$ m

L: a: $\frac{1}{a_1} = \frac{1}{F_1} - \frac{1}{b_1}$ $\Rightarrow$

$\Rightarrow a_1 = \cancel{1{,}1}$ m

$-6{,}530$ m

(7)



Fransar ekvidistanta

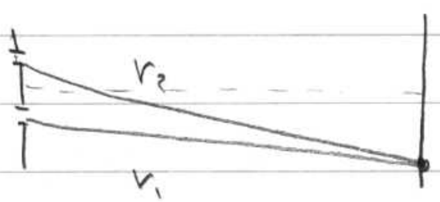
$\Delta x = 800\ \mu m \Rightarrow x_1 = 800\ \mu m$

a) int. min då : $a \sin\theta = m\lambda$, $m = 0, 1, 2 \ldots$

θ litet $\Rightarrow$ $\sin\theta \approx \frac{x}{L}$

$a\frac{x_1}{L} = 1 \cdot \lambda \Rightarrow \underline{a} = \frac{\lambda L}{x} = \underline{1{,}5\ mm}$

b) (1) utan glasplatta

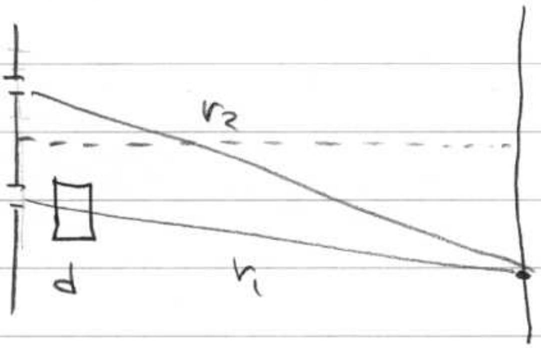


skillnad i väg:
$a \sin\theta = r_2 - r_1$

centralmax då $\Delta\varphi = 0$

Alltså, då $r_1 - r_2 \Rightarrow \theta = 0$

② med glasplatta



Optiska vägarna:

$\ell_2 = r_2$

$\ell_1 = dn + (r_1 - d)$

$d = 50{,}0\ \mu m$, $n = 1{,}58$

Centralmax då $\ell_2 = \ell_1$

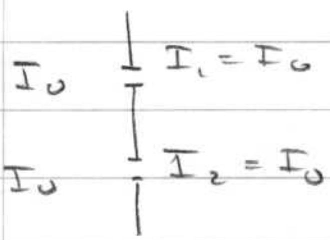
$\Rightarrow dn + r_1 - d = r_2$

$$d(n-1) = r_2 - r_1 = a\sin\theta \approx a\frac{x}{L}$$

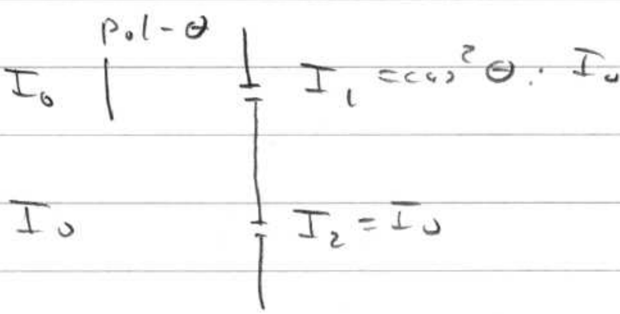
$$\Rightarrow x = \frac{Ld(n-1)}{a} = 3{,}9\ mm$$

(så mycket förskjuts mönstret)

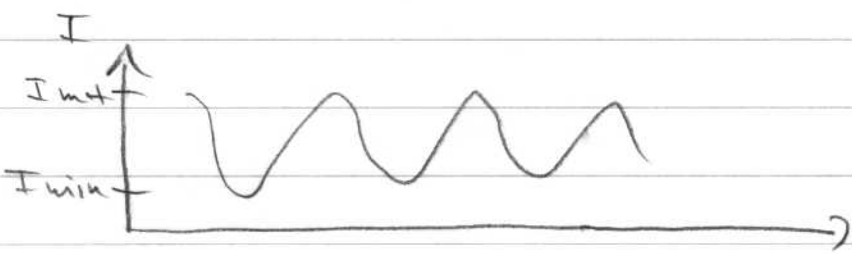
c) Före



Efter:



Visibilitet: $V = \dfrac{I_{max} - I_{min}}{I_{max} - I_{min}} = 0{,}9$



$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\delta$$

Max för $\cos\delta = +1$

Min för $\cos\delta = -1$

$$\Rightarrow I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = I_0\left(\cos^2\theta + 1 + 2\cos\theta\right)$$

$$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2} = I_0\left(\cos^2\theta + 1 - 2\cos\theta\right)$$

$$V = \frac{4\cos\theta}{2(\cos^2\theta + 1)} \quad \Rightarrow \cos^2\theta + 1 = \frac{2}{V}\cos\theta$$

$$\cos^2\theta - \frac{2}{V}\cos\theta + 1 = 0$$ Andragradare!

$$\cos\theta = \frac{1}{V} \pm \sqrt{\frac{1}{V^2} - 1} = \frac{10}{9} \pm \sqrt{\frac{100}{81} - 1} =$$

$$= 1{,}11 \pm 0{,}48$$

$$= \cancel{1{,}6} \quad \text{eller} \quad 0{,}63$$

$$\cos\theta = 0{,}63 \Rightarrow \theta = 51^\circ$$