Statistical Physics PhD course 2011 Home examination

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I.   MEAN FIELD THEORY FOR THE XY MODEL

Problem 1 out of 6

Put up a mean-field theory for the XY model in the presence of a magnetic field. Find the self-consistency equation for the magnetization vector $\vec{m}$. Find the critical temperature in zero external field, and the critical exponent for the order parameter. Discuss what happens in the case of a nonzero magnetic field.

Hint: The integral over $\exp(\cos\theta)$ is a Bessel function I_n. You can find it in a book of tables, *e.g.,* Beta or Abramowitz and Stegun, "Handbook of mathematical functions", or try `http://www.wolframalpha.com` . Make use of its small-argument expansion. Although not necessary, it might be helpful to sketch the function in Matlab.