Cold atoms 3: Optical lattices

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Today's lecture

States that are *not* simple Bose-Einstein condensates:

- Bose glass
- Mott insulators

Optical potentials

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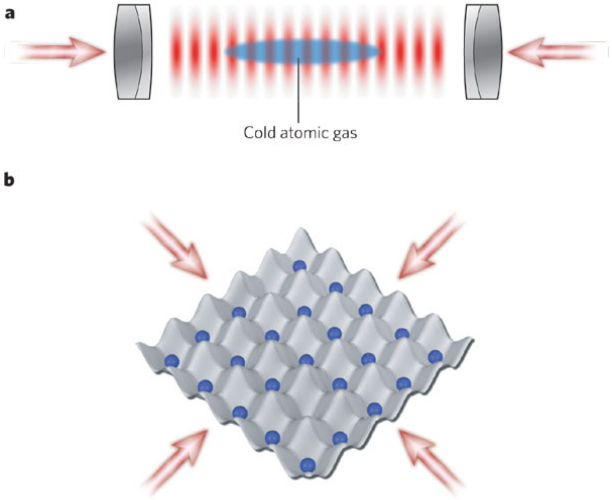
Second-order perturbation theory:

$$V(r) \propto |E(r)|^2 \propto I(r)$$

I is the irradiance

Standing wave



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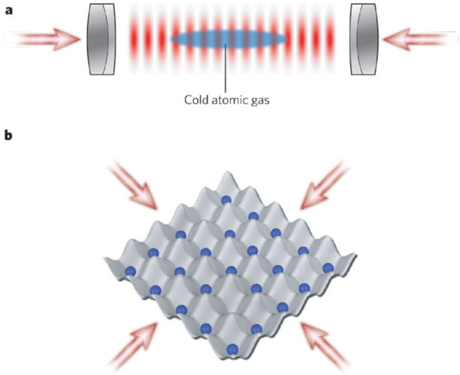
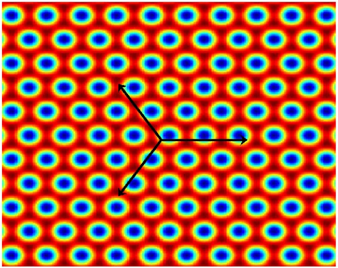
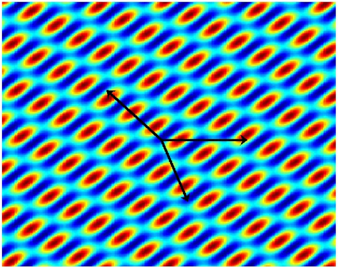


Optical lattice: $V(r) = V_{01} \cos \vec{k_1} \cdot \vec{r} + V_{02} \cos \vec{k_2} \cdot \vec{r}$

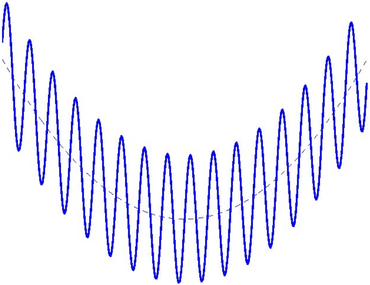
Optical lattices

Generally: Several laser beams create *interference patterns* in 1, 2, or 3 dimensions.





Lattice plus trap



Optical lattice: periodicity
Magnetic trap: Confinement

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- Therefore ideal to study *disorder* – in a controlled way!
- Disorder: Of interest in solid-state systems (impurities, lattice defects)
- Fundamental interest: Quantum mechanics + random potential
- New quantum phase: The Bose glass

Disorder with optical potentials

Alternative 1: Laser speckle
Light diffracted through roughened glass

gives a random pattern

Disorder with optical potentials

Alternative 2: Quasiperiodic potential

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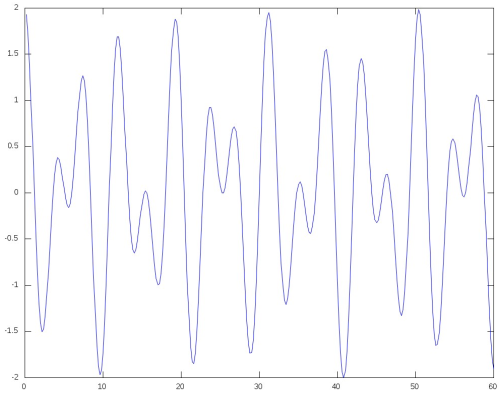
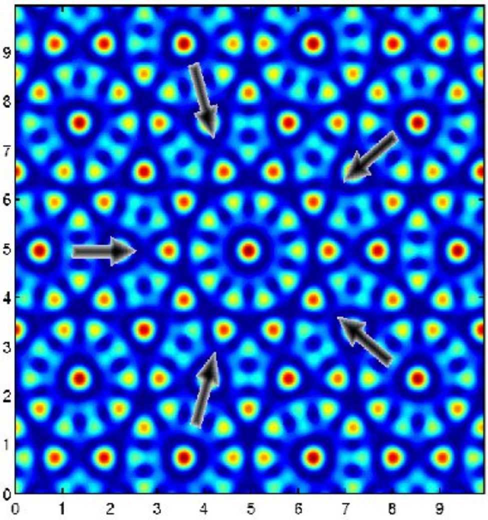
(In practice: make sure that k_1/k_2 is not close to a ratio of small integers such as 2/3 or 1/4.)
Popular choice: The golden ratio $k_1/k_2 = (1 + \sqrt{5})/2$

Quasiperiodic potential in 1D



Quasiperiodic potential in 2D



Anderson localization

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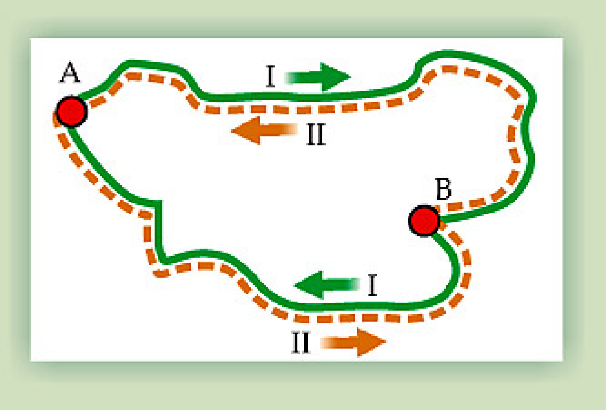
Periodic potentials: Bloch waves $\rightarrow$ transport.
Non-periodic potentials: Interference effects $\rightarrow$ localization.

Anderson's argument



Constructive interference between a path and its reverse.

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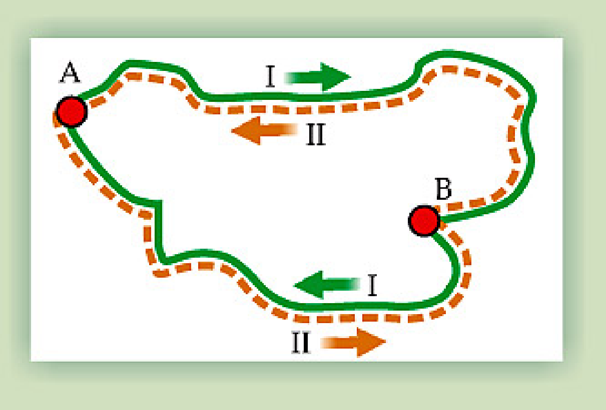
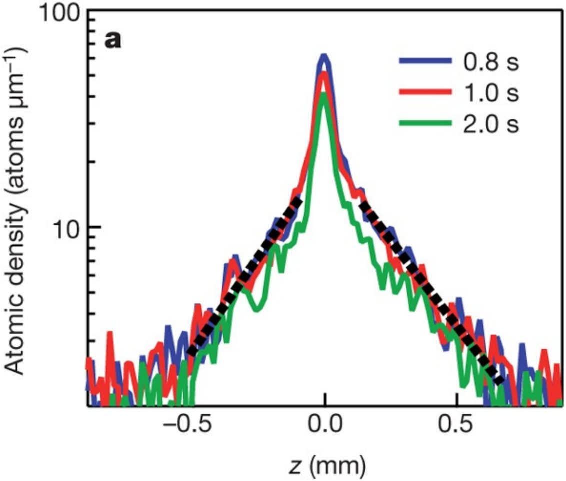


Constructive interference between a path and its reverse.
If not periodic, then *no* constructive interference for *other* paths – only those that lead back to point of origin!

Exponential localization

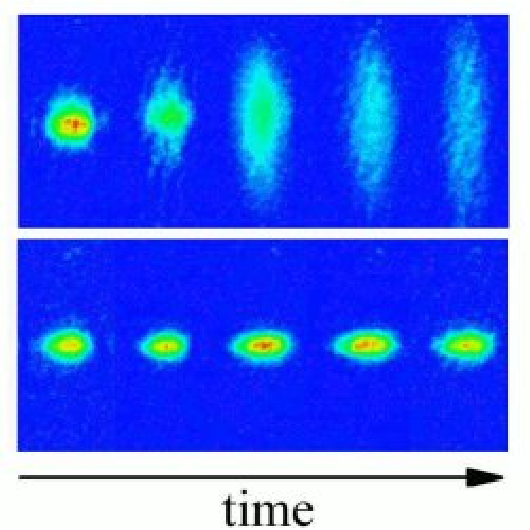


Wavefunctions decay exponentially at long distances: Localized eigenstates.

Observation of Anderson localization

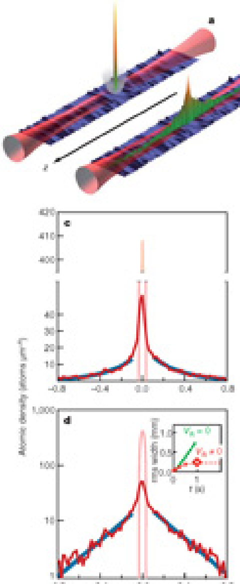
- Not in solids: Mean free path too short
- Must construct artificial system in order to see A.L.
- Light: 1997
- BEC: 2008
- Very dilute BEC: neglect interactions.
- AL is a single-particle phenomenon.

Experiment, Florence



Roati *et al.*, Florence. Quasiperiodic potential in 1D

Experiment, Paris



Billy *et al.*, Paris.
Speckle potential in 1D

Bose Glass

What about interacting Bose gases?

Bose Glass

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- A quantum phase was predicted, the “Bose glass” phase.
- For the experts: Gapless but not superfluid.
- Loosely speaking: System is broken up into several small BEC, whose phases are mutually random

Bose Glass

Define the particle-particle correlation function

$$g(r, r') = \int dr_2 \cdots dr_N \psi^*(r, r_2, \ldots, r_N)\psi(r', r_2, \ldots, r_N)$$

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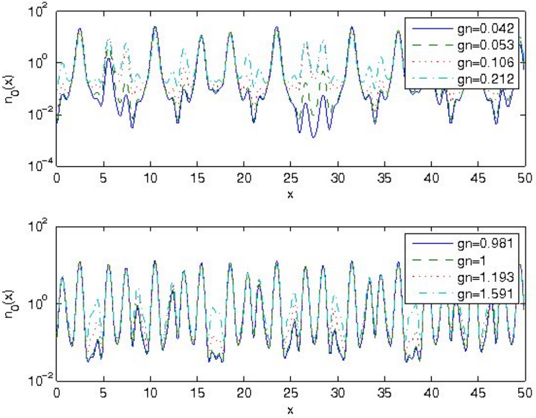
BEC: $g(r, r') \sim \mathrm{const}$ at large $r - r'$

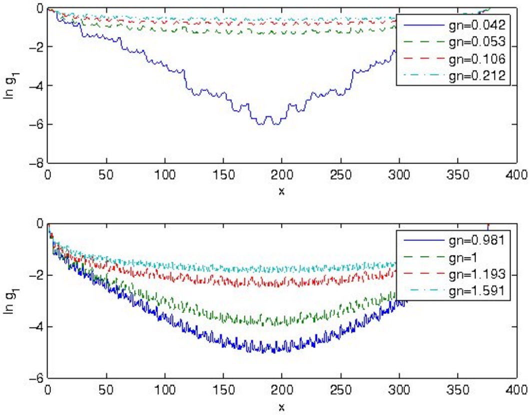
Bose glass: $g(r, r') \sim e^{-|r-r'|}$ exponential decay

Bose glass: Density



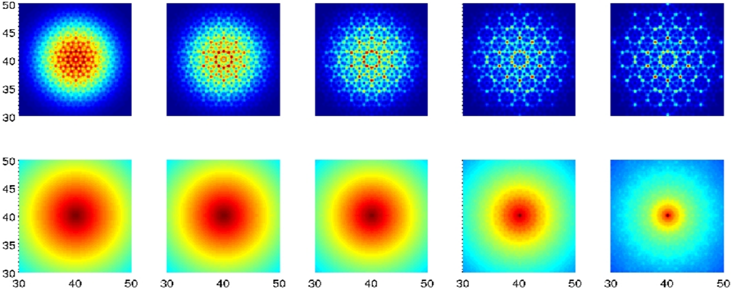
Cetoli and Lundh, 2010

Bose glass: Correlations



Cetoli and Lundh, 2010

Bose glass in 2D



Cetoli and Lundh, to be published

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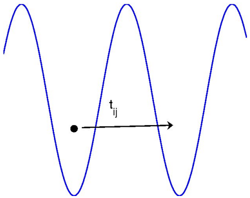

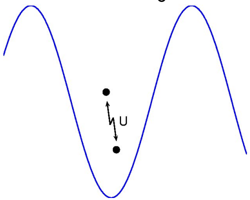

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Put all this together into Hamiltonian:

$$H - \mu N = -t \sum_{<i,j>} a_i^\dagger a_j + \frac{U}{2} \sum_i a_i^\dagger a_i^\dagger a_i a_i - \mu \sum_i a_i^\dagger a_i$$

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- Superfluid: When kinetic energy dominates, J/U not small.
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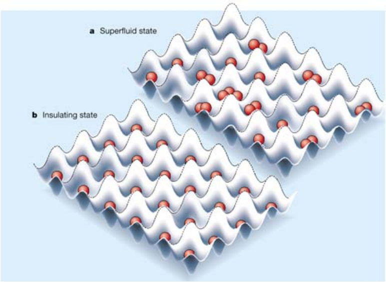

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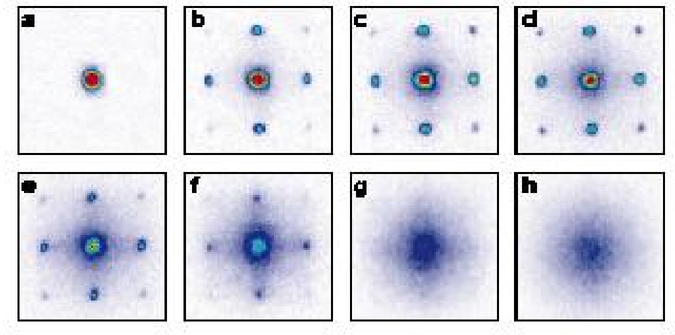
- Takes place at zero temperature.
- Driven by quantum fluctuations, not thermal.
- Can be described by:
  - Monte Carlo
  - DMRG
  - Mean-field theories
  - Strong-coupling expansion
  - some more
- Mostly heavy numerical computations.

Key observation: BEC is coherent – will give interference fringes. Mott insulator is not.
Periodic system ⇒ peaks at reciprocal lattice vectors!



First experiment by Greiner et al., 2001

Trapped systems

$$H - \mu N = -t \sum_{<i,j>} a_i^\dagger a_j$$
$$+\frac{U}{2} \sum_i a_i^\dagger a_i^\dagger a_i a_i + \sum_i \left(\frac{1}{2}\omega^2 r_i^2 - \mu \right) a_i^\dagger a_i$$

Superfluid and Mott insulating regions coexist

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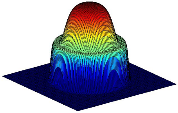
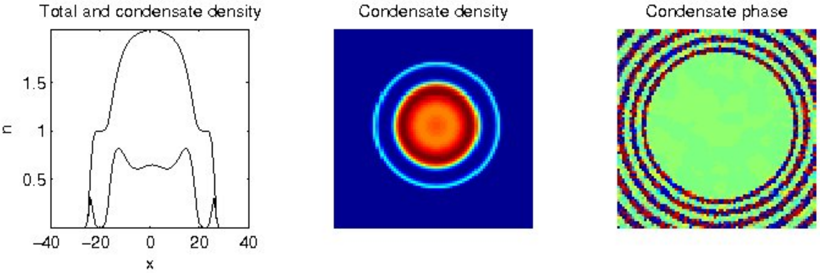




Density – condensate density – condensate phase
Note that a *local-density approximation* applies.

Rotation in optical lattice

Can we have vortices in an optical lattice?

Rotation in optical lattice

Can we have vortices in an optical lattice?
Techniques:

- Rotate the actual lattice
  - Mechanically rotating mask
  - Interference pattern modulated in time
- Artificial gauge fields

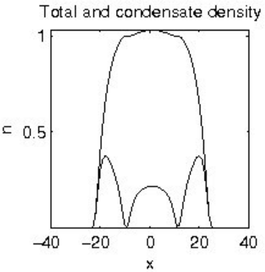
What I found

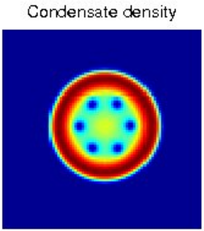
Two limits:

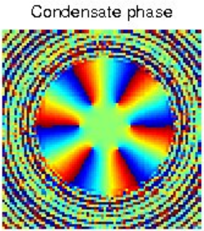
1. Close to the Mott transition; vortex has Mott insulating atoms in the core

2. Further from the Mott transition; vortex core is smaller than the lattice spacing. Vortex sits between the lattice sites where density vanishes

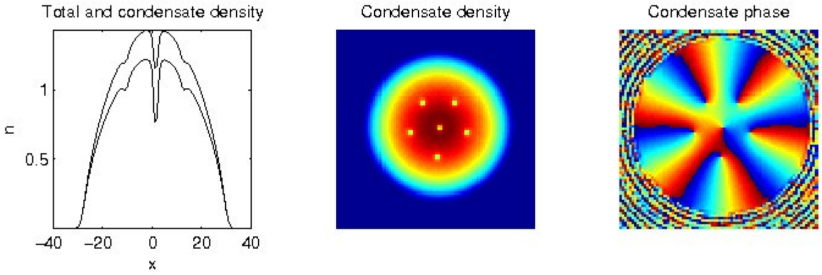
Vortices with Mott cores







Interstitial vortices



End of lecture 3

Thank you for your attention