Two Studies of Two-Dimensional Vortex Glass Models

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References:

[1.] Petter Holme and Peter Olsson, *A Zero-Temperature Study of Vortex Mobility in Two-Dimensional Vortex Glass Models*.

[2.] Petter Holme, Beom Jun Kim, and Petter Minnhagen, *Two Distinct Superconducting Phases in the Two-Dimensional Random Gauge XY Model*.

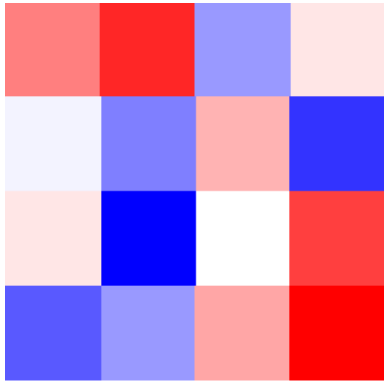
CONTENTS

DEFINITION OF THE MODELS

$$\mathcal{H} = \sum \cos(\theta_i - \theta_j - A_{ij} - \Delta_{ij})$$

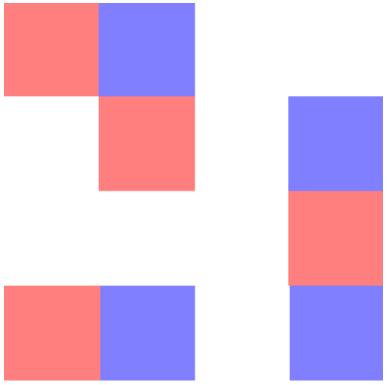
RANDOM GAUGE *XY* MODEL

$A_{ij} \in [-r\pi, r\pi)$, $0 \leq r \leq 1$. Standard *XY* gauge glass corresponds to $r = 1$.



GENERALIZED *XY* SPIN GLASS MODEL

$A_{ij} \in \{0, \pi\}$, $A_{ij} = \pi$ with probability s, $0 \leq s \leq 1$. Standard *XY* spin glass corresponds to $s = 1/2$.



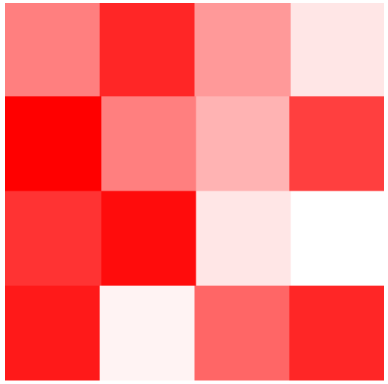
RANDOM PINNING MODEL

$$\mathcal{H} = -\frac{1}{2} \sum_{\mathbf{r} \neq \mathbf{r}'} (q_{\mathbf{r}} - f) G(\mathbf{r} - \mathbf{r}') (q_{\mathbf{r}'} - f) - \sum_{\mathbf{r}} v_{\mathbf{r}} q_{\mathbf{r}}^2 .$$

$v_{\mathbf{r}} \in [-\pi, \pi)$ is a random variable. The vorticity $q_{\mathbf{r}}$ is restricted to $\{-1, 0, 1\}$.

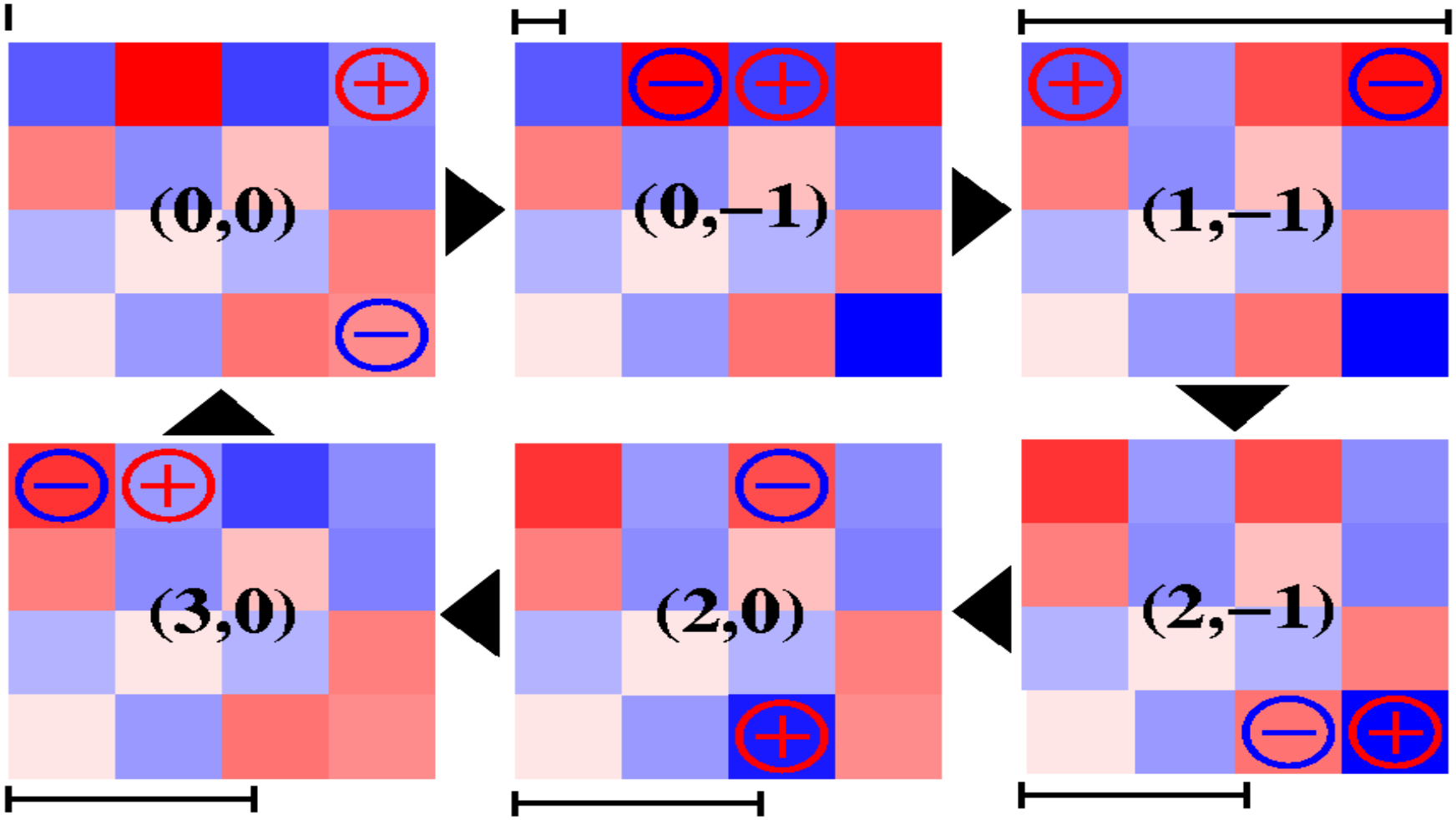
$G(\mathbf{r})$ is the lattice Green's function

$$G(\mathbf{r}) = \left(\frac{2\pi}{L}\right)^2 \sum_{k \neq 0} \frac{1}{4 - 2\cos k_x - 2\cos k_y}$$



ZERO TEMPERATURE: THE BARRIER FOR A 2π PHASE SLIP, V_L

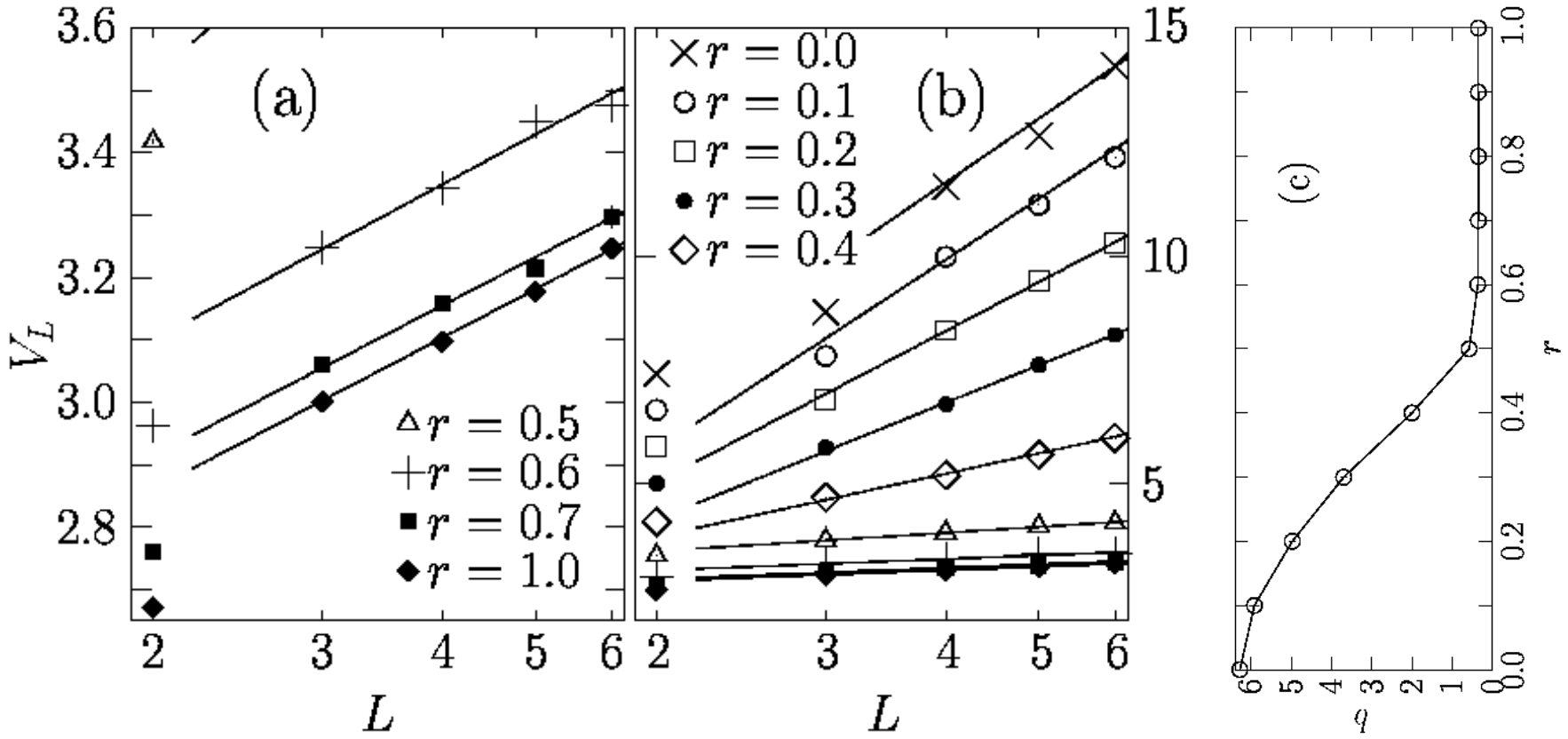
V_L is the lowest energy cost for a vortex charge to move across the system.



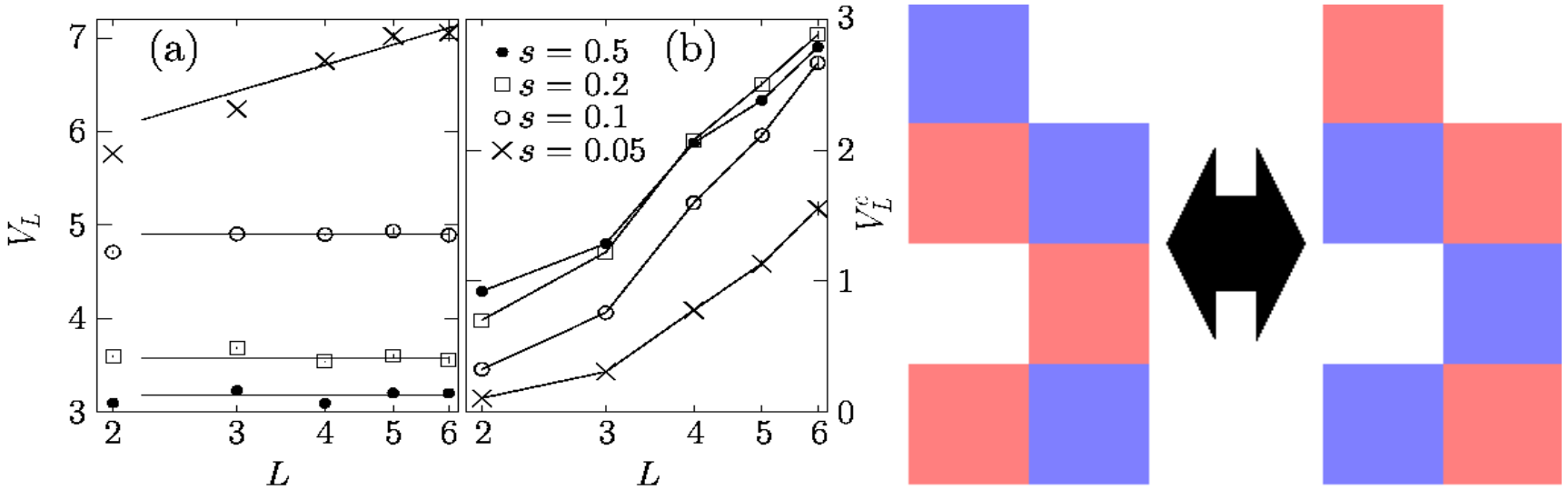
ALGORITHM FOR FINDING V_L

1. Generate $4L^2$ configurations by applying the $4L^2$ possible dipole excitations to the current configuration.
2. Calculate the energy of each such configuration and put them in a sorted list, lowest energy first, together with their polarization relative to the ground state.
3. Take the first (lowest energy) configuration from this list to be the new current configuration.
4. If this configuration has already been encountered, but with a different polarization such that $\Delta P = (\pm L, 0)$ then we are done. Otherwise, go to step 1.

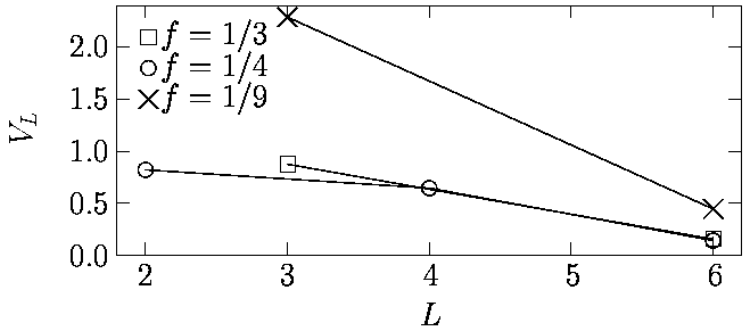
RANDOM GAUGE *XY* MODEL



GENERALIZED *XY* SPIN GLASS MODEL



RANDOM PINNING MODEL

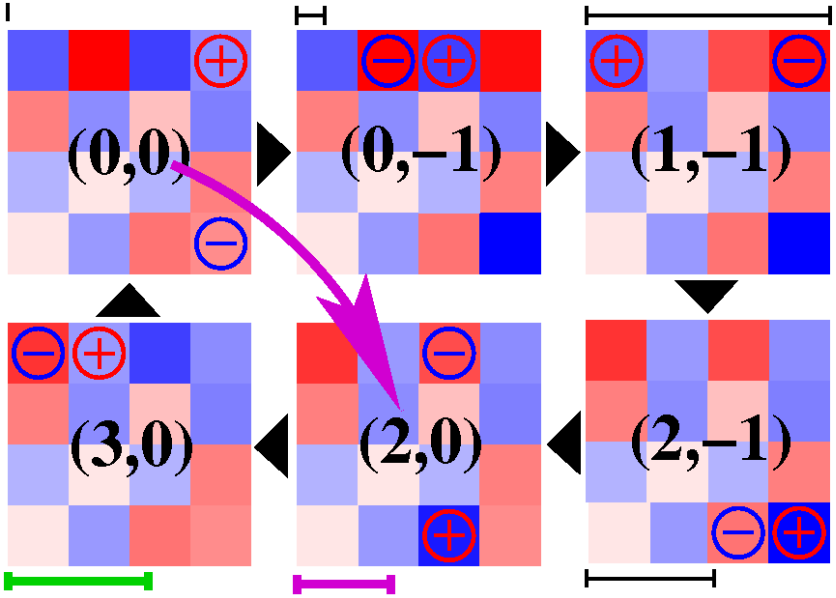


ZERO TEMPERATURE: V_L VS DOMAIN WALL ENERGY. ERGODICITY BREAKING.

(Best Twist) Domain wall energy:

$$\min E(\Delta_0) - \min E(\Delta_0 + \pi)$$

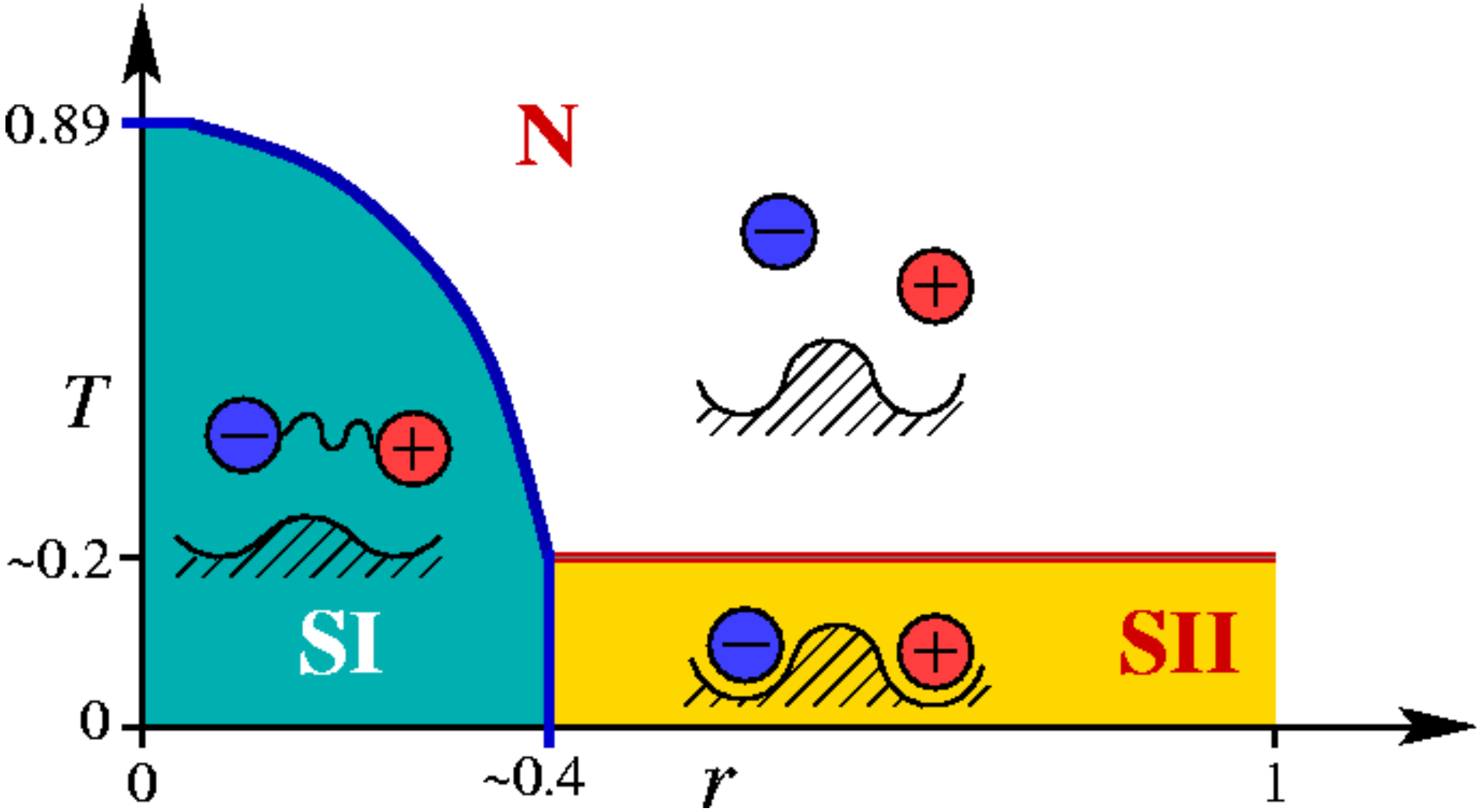
where Δ_0 is the twist that gives the (twist space) ground state.



To get to the $\Delta_0 + \pi$ ground state might involve intermediate barriers. It may also *not* be on the V_L-path.

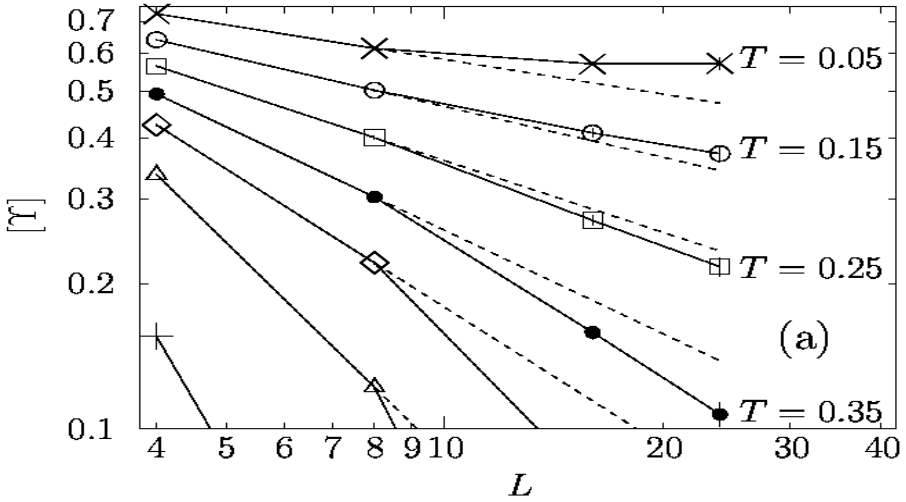
- Neighboring points in the twist free-energy landscape can be widely separated in configuration space.
- Domain wall energy has no direct connection to the barrier for vortex motion in disordered models unless quantum tunneling of vortices is assumed.
- $V_L \to \infty$ as $L \to \infty$. $\Longrightarrow$ The system is locked into a subspace of the configuration space. $\Longrightarrow$ Ergodicity is broken.
- Random gauge XY model: The drastic change at $r \approx 0.4$ (found to be a phase boundary in DWE studies, i.e. assuming quantum tunneling), suggests a low-r phase of pair-bound vortices and a high-r phase of vortices pinned by the disorder.

Phase Diagram for the 2D Random Gauge *XY* Model

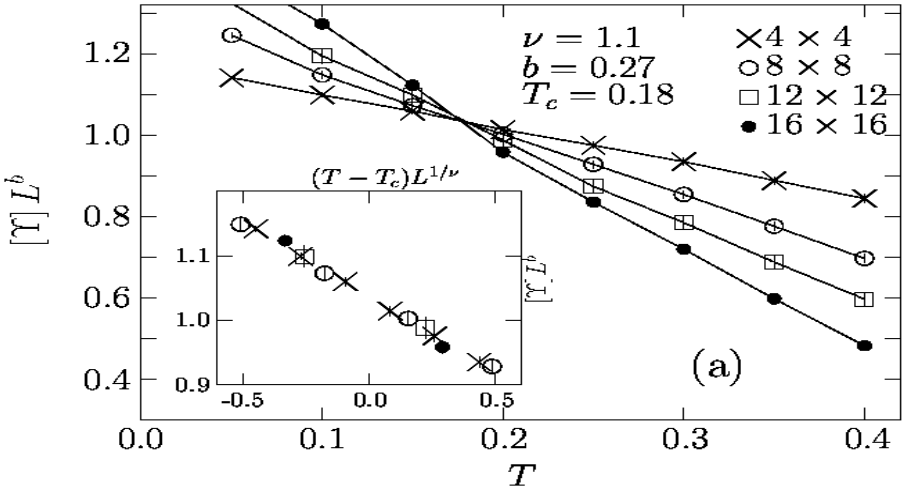


FINITE TEMPERATURE: NORMAL ↔ SUPERCONDUCTING TRANSITIONS

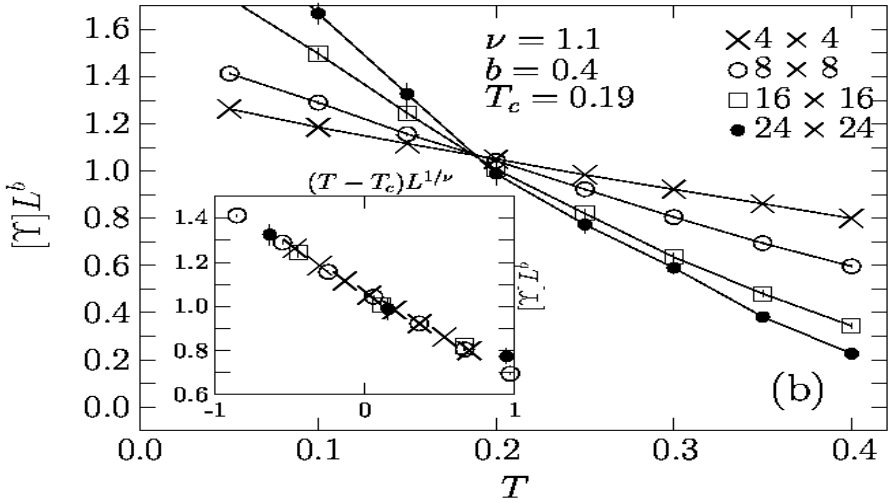
Finite helicity modulus for gauge glass:



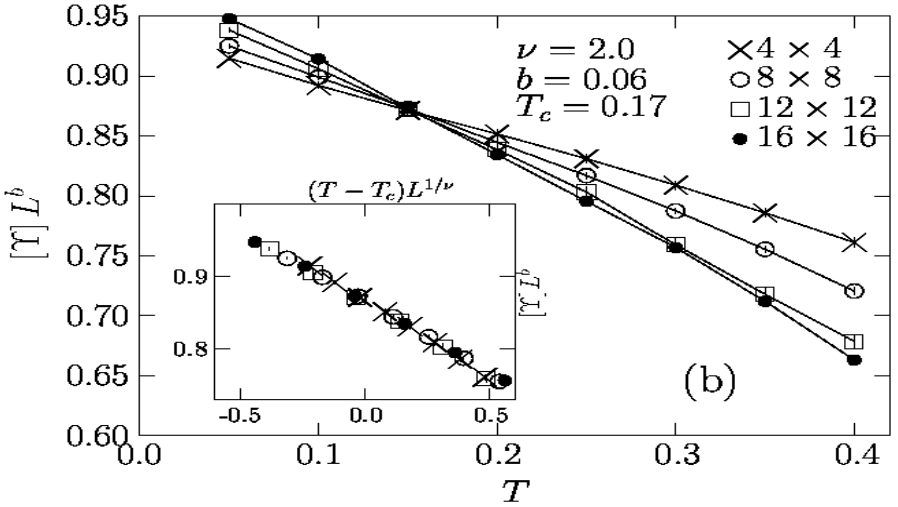
FSS of helicity modulus for $r = 0.5$:



FSS of helicity modulus for gauge glass:

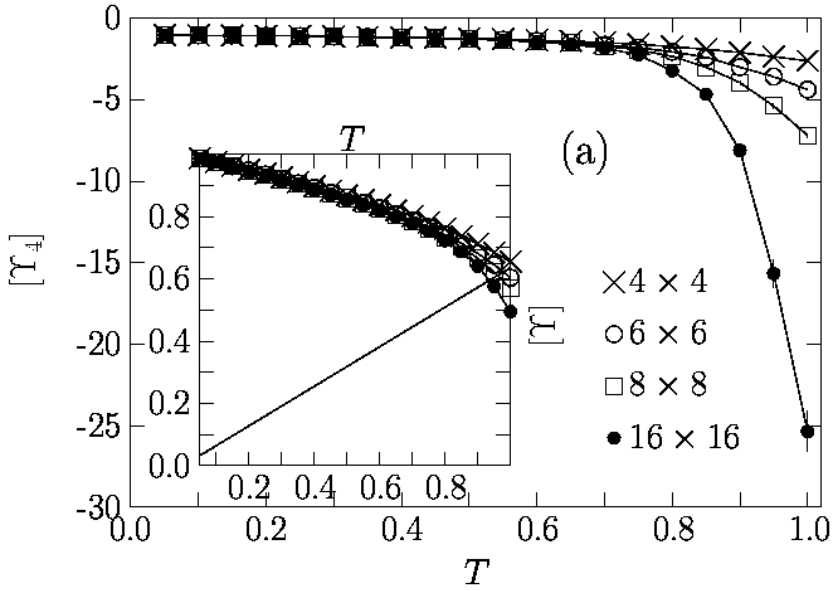


FSS of helicity modulus for $r = 0.4$:

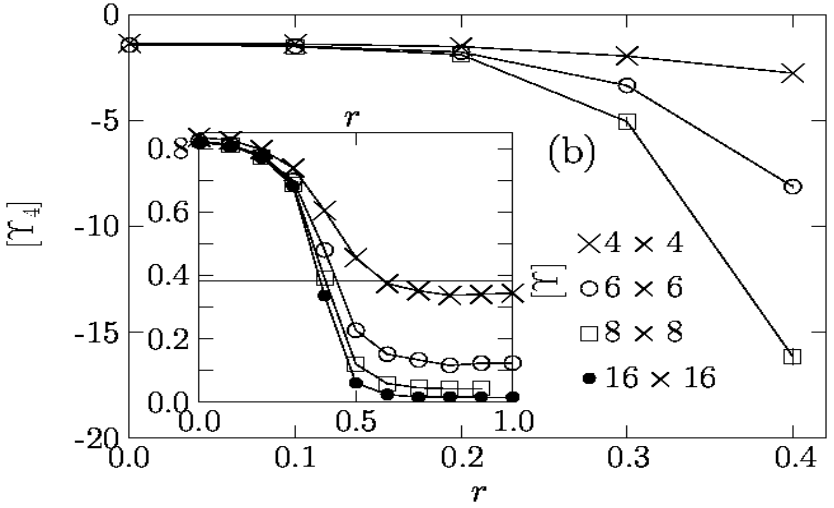


FINITE TEMPERATURE: PAIR-BOUND ↔ DISORDER-PINNED TRANSITION

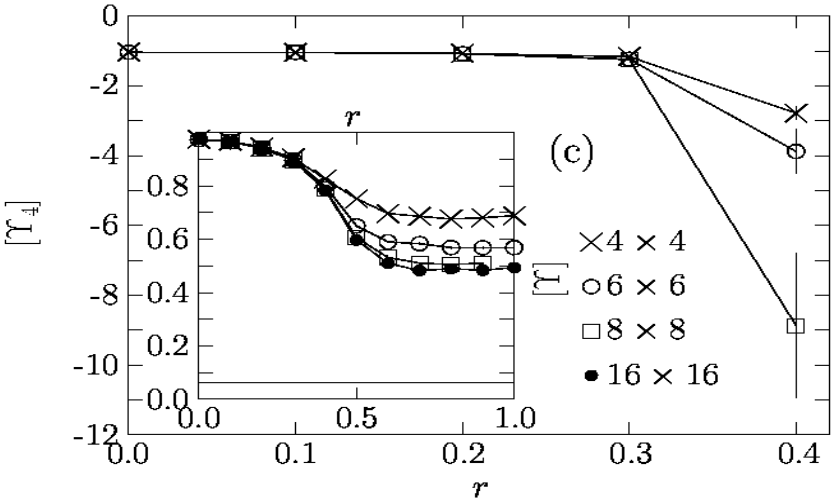
Fourth order modulus vs T for standard XY model ($r = 0$):

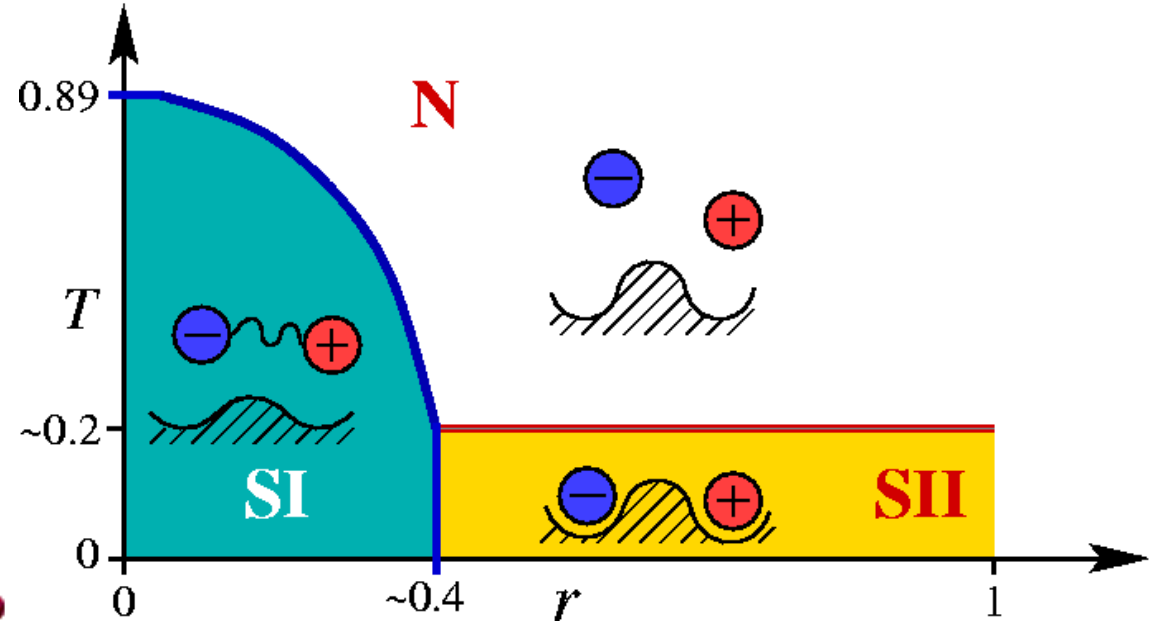


Fourth order modulus vs r for $T = 0.6$:



Fourth order modulus vs r for $T = 0.1$:





- $T_c = 0$ but possibly $T_c^c > 0$ for XY spin glass model.
- No finite-T superconducting phase in the random pinned model.