The structure of an Internet dating community (Introducing bipartivity)

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SOCIAL NETWORKS

- from group psychology . . .
- . . . to graph theory
- most intangible of networks
- epidemiology
- anthropology
- psychology
- defense

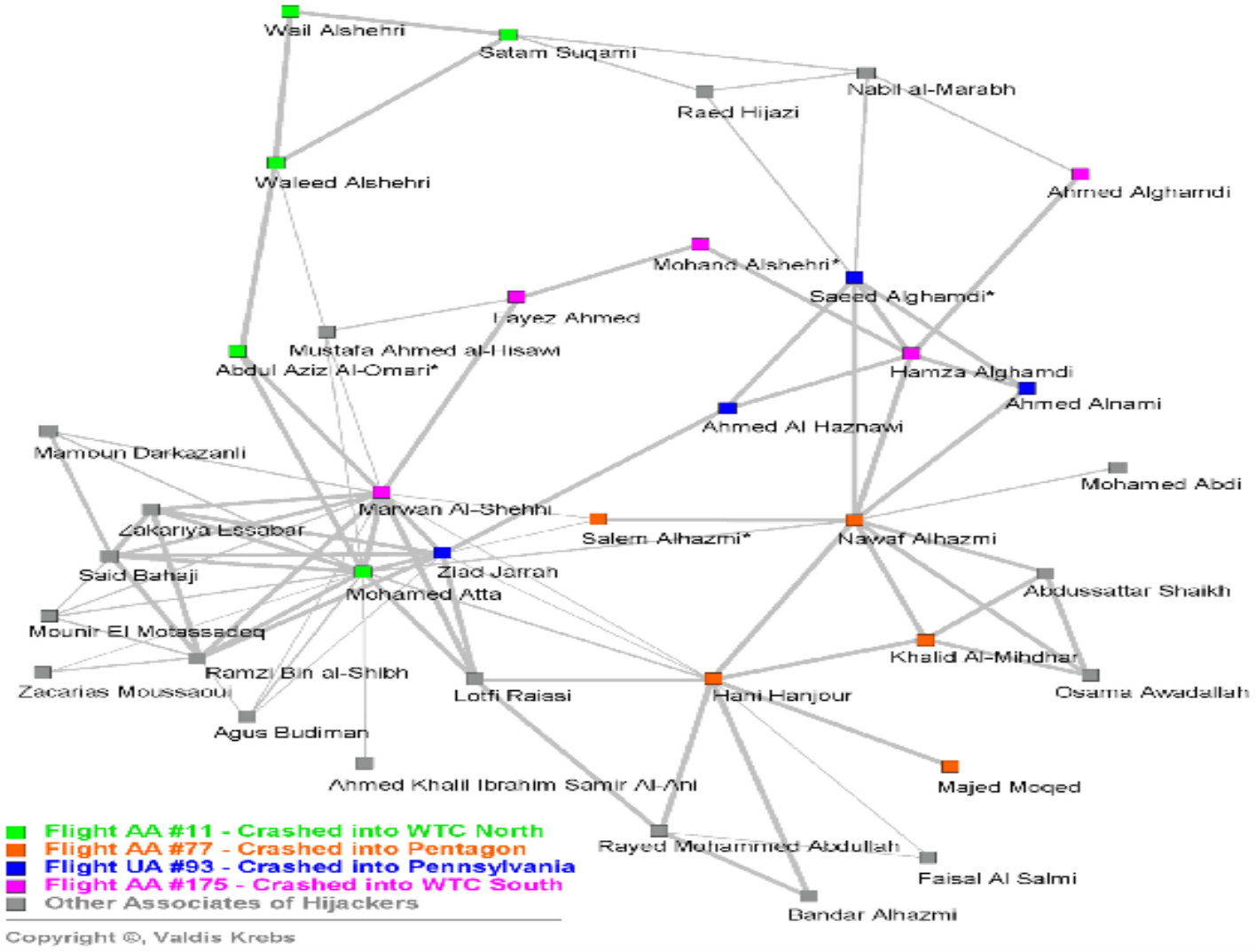


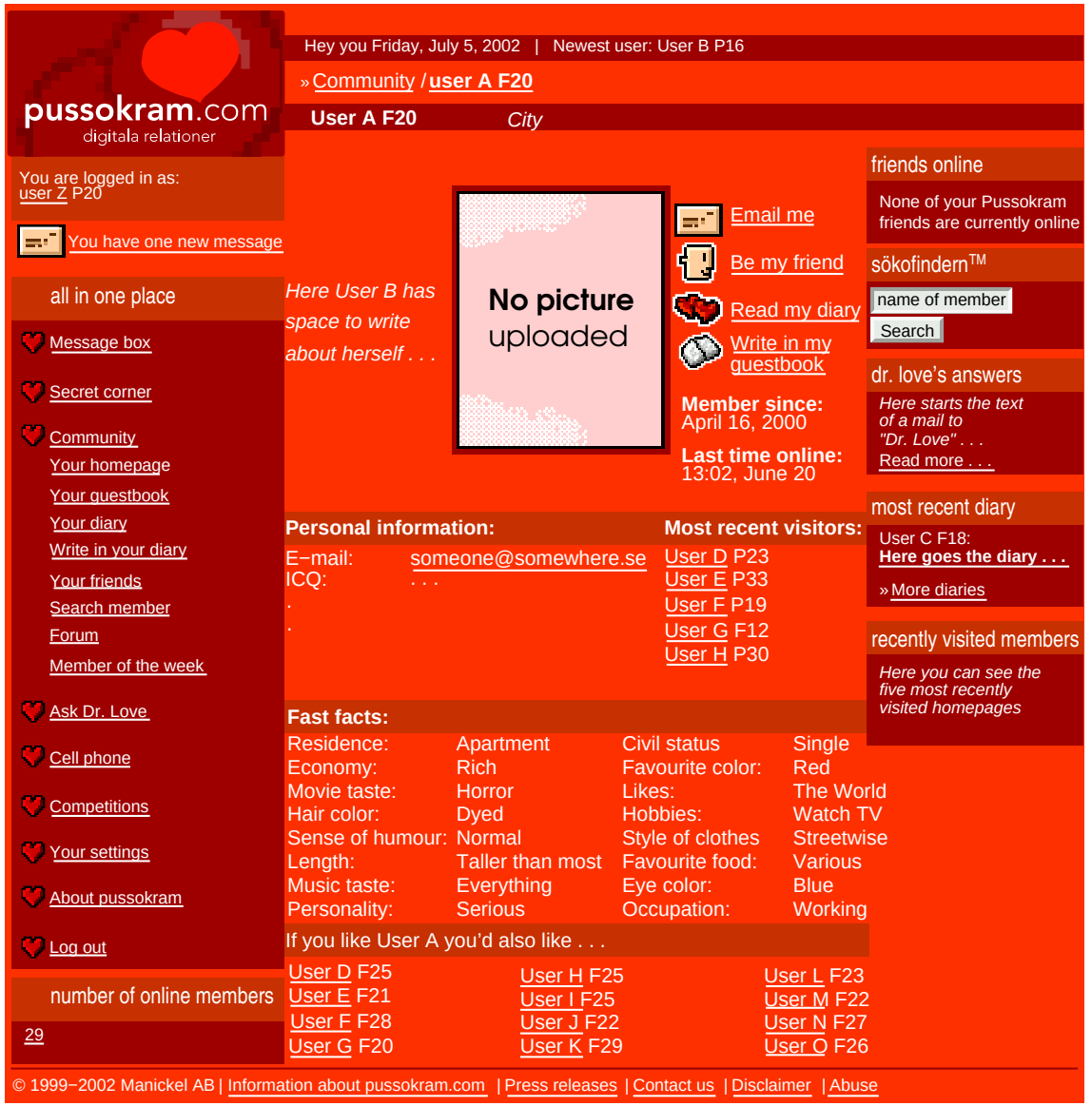
Figure 4. Hijacker's Network Neighborhood

WHY INTERNET COMMUNITIES

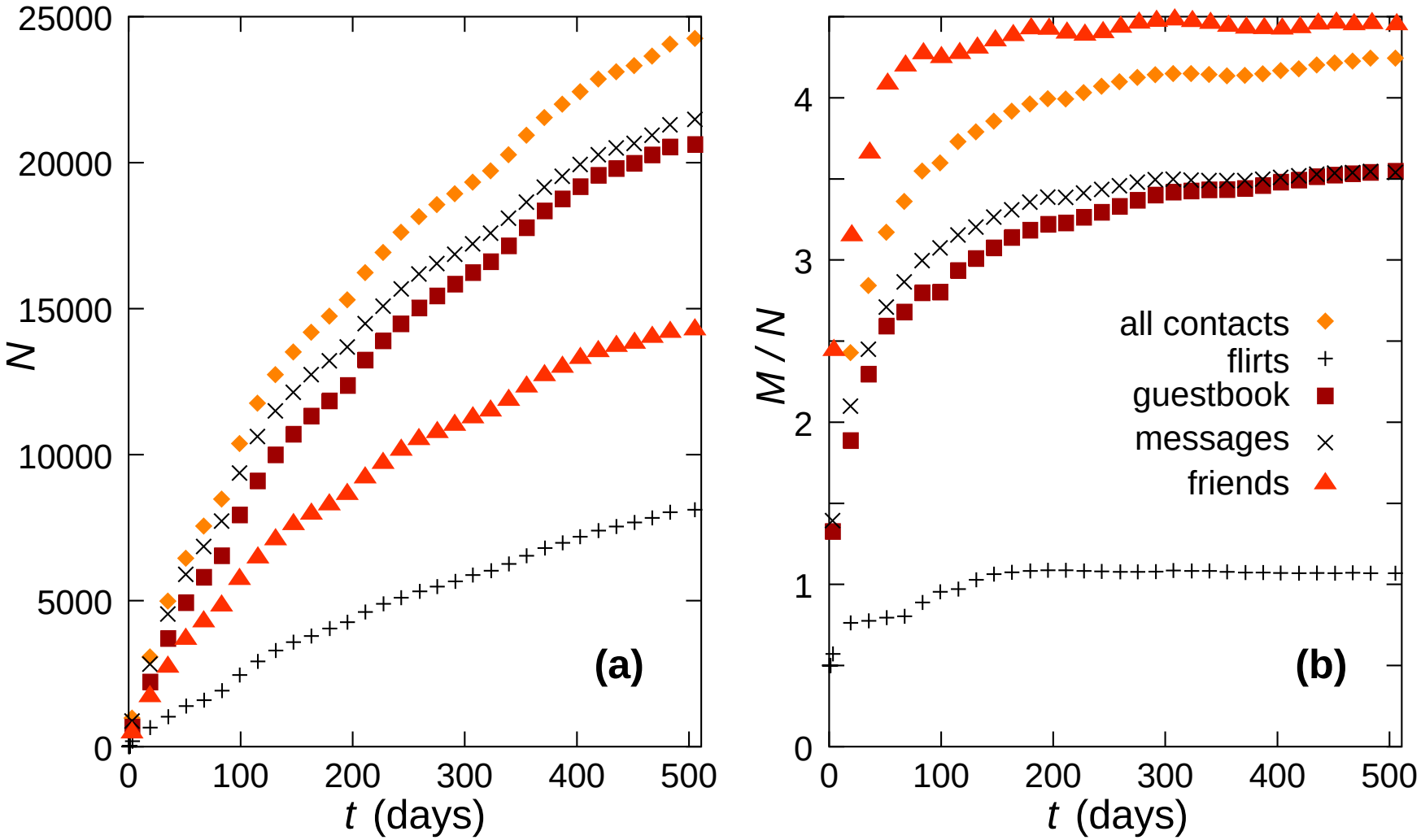
- large size (nice for statistical mechanics style approach)
- precise timing resolution
- formation more close to real acquaintance network than other methods
- interesting *per se*

about pussokram.com:

- ~ 130 000 users
- ~ 70% female
- average age 21 yrs (?)



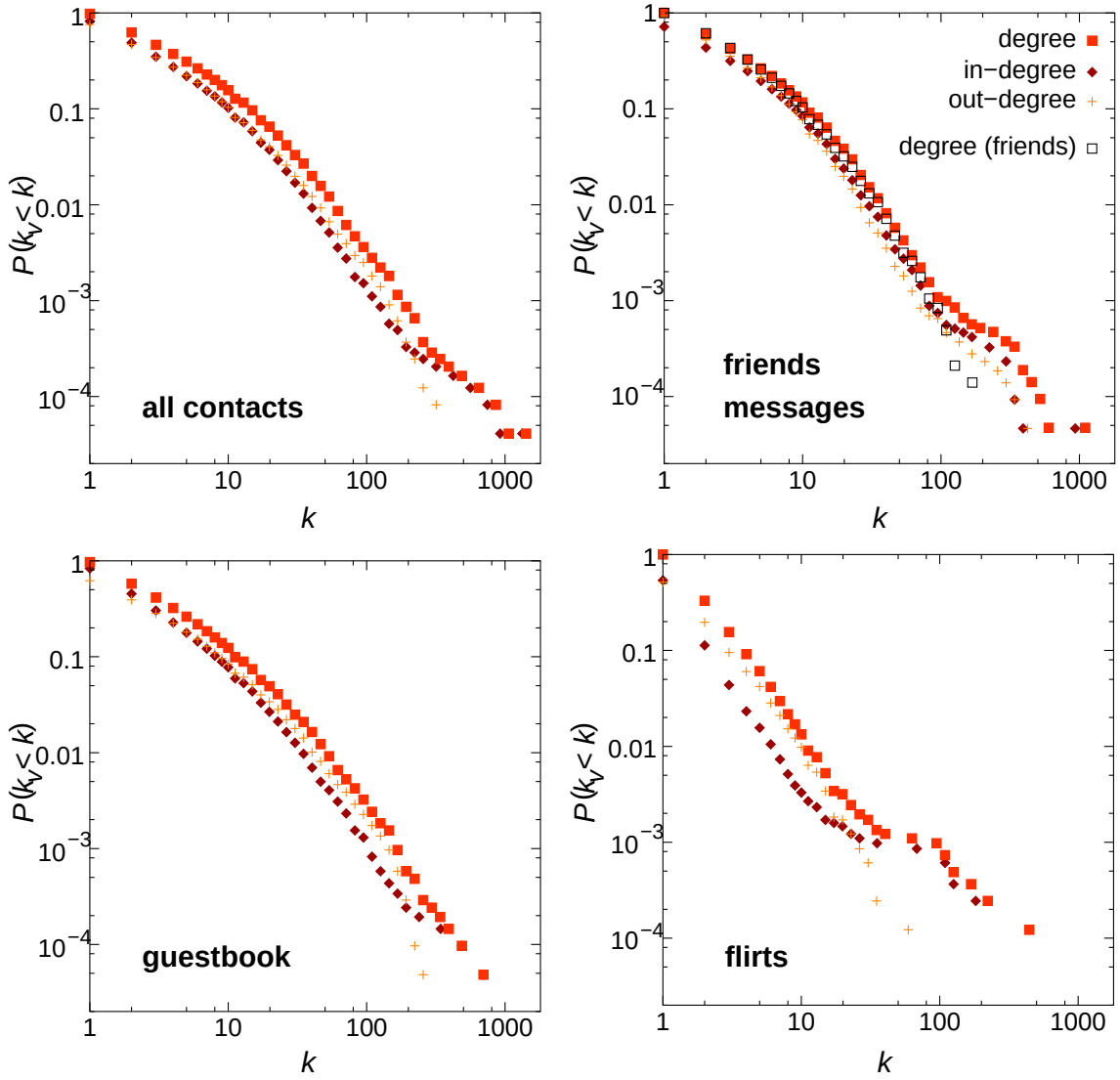
TIME DEVELOPMENT



DEGREE DISTRIBUTION

. . . quite scale-free, as seen in e.g. web of sexual contacts.

(There has to be a cut-off but its nowhere to be seen)

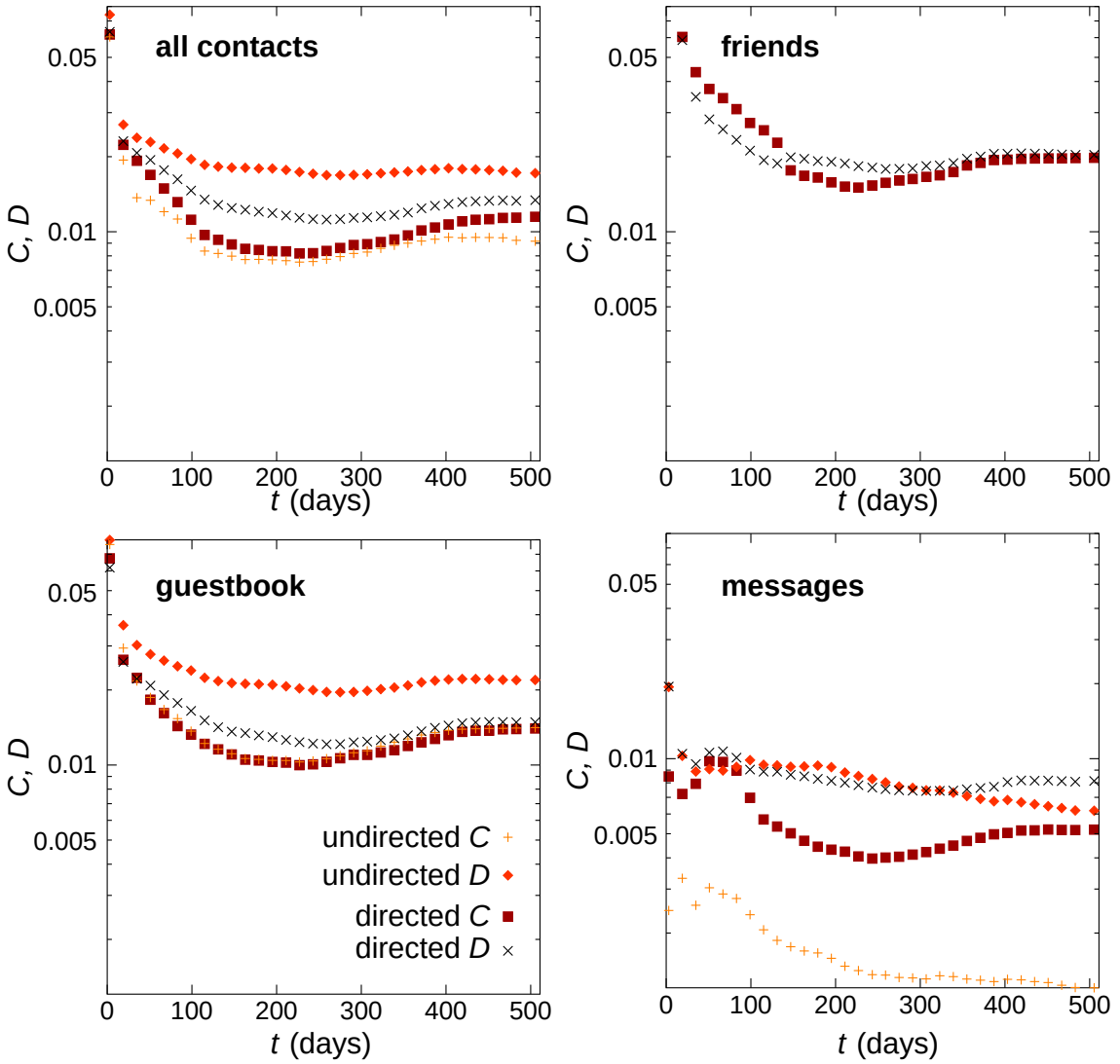


Density of short circuits

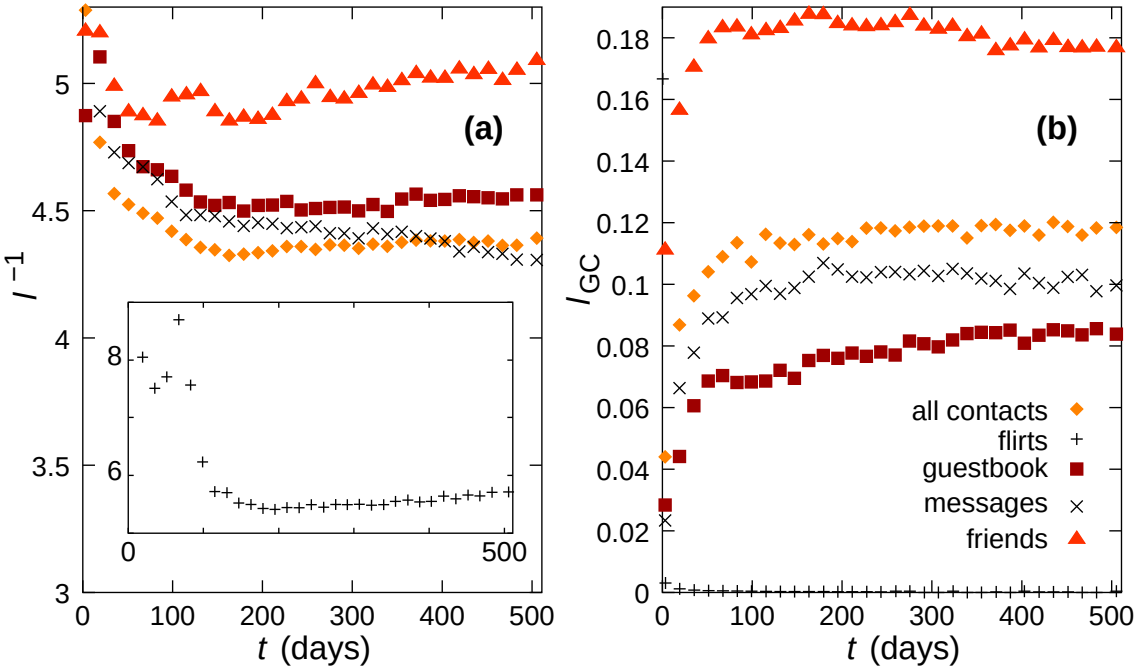
C—density of triangles,
D—density of squares.

High C in acquaintance networks.

$C > D$ in other empirical network data. Here $D > C$ since most contacts are romantic & heterosexual.



CHARACTERISTIC PATHLENGTH



$$l_{\mathrm{GSCC}} = \frac{1}{|A_{\mathrm{GSCC}}|} \sum_{(u,v)\in A_{\mathrm{GSCC}}} d(u,v)$$

$$l^{-1} = \frac{1}{M} \sum_{(u,v)\in A} \frac{1}{d(u,v)}$$

BIPARTIVITY . . .

In romantic networks there's another structure from hetrophilous interaction, bipartivity—the network is closer to bipartite than a random network. How can this structure be measured in a general graph?

Bipartivity measures Supposing a strong hetrophilous interaction ⇒

bipartivity = the fraction of unfrustrated edges in the ground state configuration of the antiferromagnetic Ising model:

$$H = \sum_{(u,v)\in E} S_u S_v$$

where $S_x \in \{-1, +1\}$ is a number assigned to every vertex.

Disadvantages with this measure:

- NP-complete optimization problem

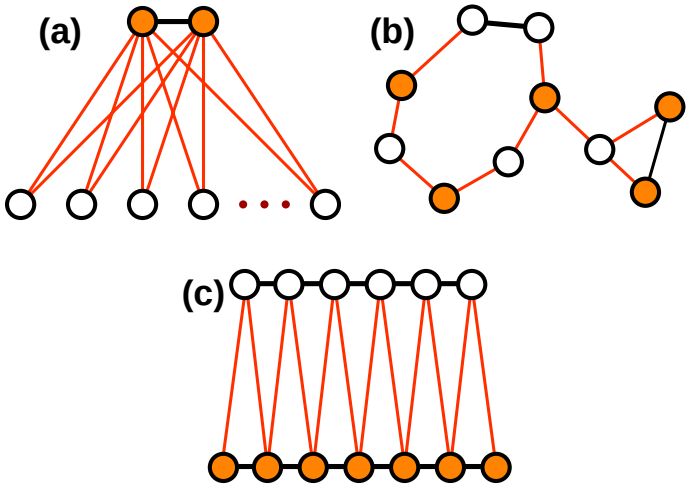
Solution: Calculate approximate value by simulated annealing.

or

Construct a heuristic measure calculable in polynomial time . . .

Base the measure on odd circuit count . . .

To take into account . . .



(a) A highly bipartite graph can have many odd circuits.

(b) Not only triangles contribute.

(c) If many circuits are present, short ones are more important.

- Let C_n be the set of odd circuits of length $\leq n$.
- Let $\Sigma(C_n)$ be the accumulated length of the circuits of C_n (so, for example $\Sigma(C_3) = 3$ in (b)).
- Assign the cut-off $3M$ to $\Sigma(C_n)$, and let $\hat{n}$ be the smallest n such that $\Sigma(C_n) \geq 3M$.

This can be done in $O(MC)$ (C is the total number of circuits) by Johnson algorithm. If the bound on n is realized the effective running time is $O(M^2)$.

Let $\nu(a)$ denote the number of cycles in $C_{\hat{n}}$ passing through the arc a.

1. Start with $C = C_{\hat{n}}$.
2. Sort the edges in order of ν.
3. Repeat the following while $C \neq \varnothing$:

(a) Mark the edge e with highest ν.

(b) Remove all cycles in C containing e.

(c) Recalculate ν for each edge.

Then the number of iterations m' is the assessment of M_{fr}, and we define our bipartivity measure as

$$b_2 = 1 - \frac{m'}{M} \, . \qquad (1)$$

. . . FROM OPTIMIZATION BY XMC

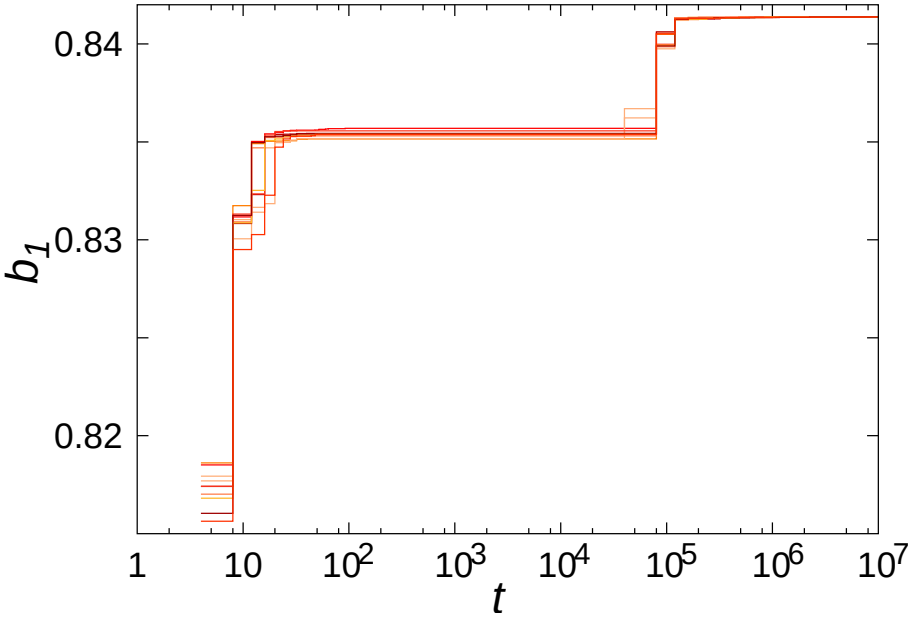
Simulated annealing: Run Monte Carlo at low temperature and save the lowest energy attained.

To make this effective we use two tricks:

- Use Exchange Monte Carlo (XMC, to avoid getting stuck in local minima).
- Quench the system (flip spin as long as the energy is lowered) from time to time. (This is done to get the really low energies.)

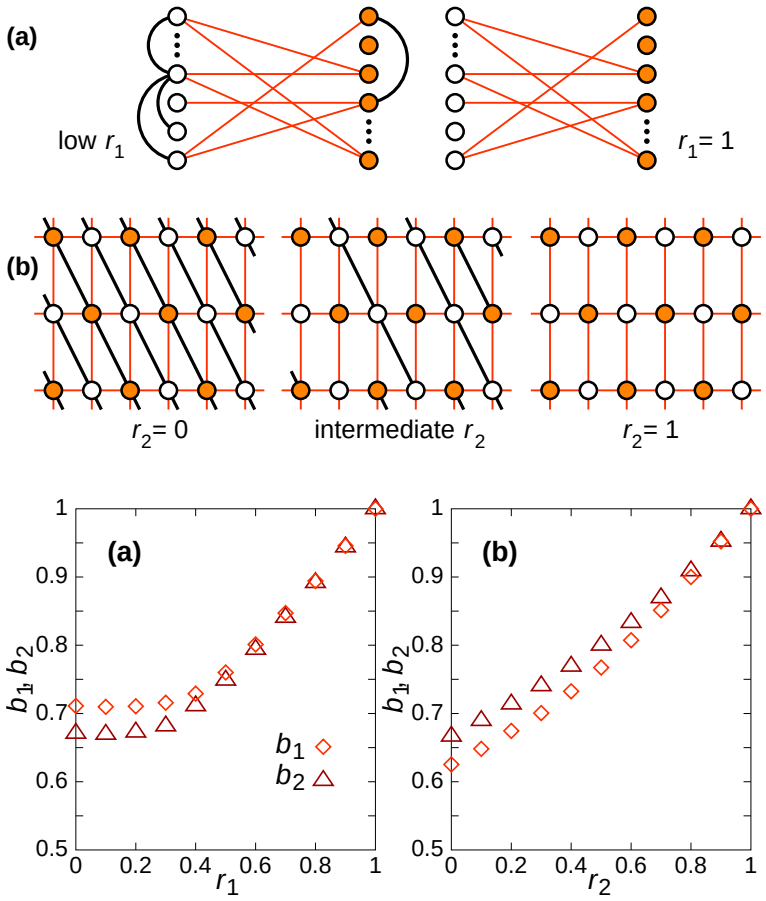
In XMC on runs the replicas of the system at several temperatures simultaneously. At certain intervals replicas at adjacent temperatures are compared and swapped if a certain condition holds.

So different replicas can travel up and down in temperature, and thus avoids getting stuck in a local minimum.



Quenches at times separated by 40 MC sweeps (giving the first jump). Exchange trials at times separated by 2×10^4 MC sweeps (giving the second jump). *So running XMC makes sense.*

Model networks



Real World networks

network	N	M	b_1^{dir}	b_1	b_2^{dir}	b_2
all contacts	29 341	174 662	–	0.860	0.948	0.928
messages	20 691	73 346	–	0.892	0.984	0.964
guestbook	21 545	76 257	0.863	0.889	0.943	0.965
nioki.com	50 259	405 742	0.842	0.855	0.956	0.975
arxiv.org	52 909	490 600	×	0.63	×	0.623
directors	7 475	48 899	×	0.549	×	0.507