Noise spectrum of a tunnel junction coupled to a nanomechanical oscillator

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A nanomechanical resonator coupled to a tunnel junction is studied. The oscillator modulates the transmission of the junction, changing the current and the noise spectrum. The influence of the oscillator on the noise spectrum of the junction is investigated, and the noise spectrum is obtained for arbitrary frequencies, temperatures and bias voltages. We find that the noise spectrum consists of a noise floor and a peaked structure with peaks at zero frequency, the oscillator frequency and twice the oscillator frequency. The influence of the oscillator vanishes if the bias voltage of the junction is lower than the oscillator frequency. We demonstrate that the peak at the oscillator frequency can be used to determine the oscillator occupation number, showing that the current noise in the junction functions as a thermometer for the oscillator.

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I. INTRODUCTION

The recent years have seen the shrinking of mechanical components to micrometer and further to nanometer size, spawning the new field of nanomechanics.[1,2] Manufacturing techniques borrowed from semiconductor chip manufacturing (see e.g. Ref. 3) as well as bottom up approaches utilizing nanotubes[4] make it possible to produce nanomechanical resonators with resonance frequencies presently demonstrated up to 1 GHz. Nanomechanical resonators are of interest for several reasons. In proposed tests of the limits of quantum mechanics one would like to investigate the decoherence behavior of superpositions of macroscopically distinct states, of e.g. a nanomechanical resonator.[5,6] Typical nanomechanical resonators contain a macroscopic number of atoms, $\sim 10^7$, making the amplitude of the basic mode a macroscopic observable. The signature of such superpositions is strongest for low oscillator occupation numbers. Presently obtainable resonator frequency is high enough to make cooling to the ground state feasible. Experimental efforts are under way to reach the first goal necessary for these measurements, cooling a nanomechanical resonator to the ground state.[7–11]

Other applications include the use of nanomechanical resonators as ultra-sensitive force detectors. Mass detection with zeptogram resolution utilizing nanomechanical resonators has been realized only recently.[12] Nanosized cantilevers can also be used to detect magnetic forces. Magnetic resonance force microscopy with high enough sensitivity to detect single nuclear spins would make three dimensional imaging of proteins possible. Detection of a single electron spin using a nanomechanical cantilever has already been demonstrated.[13] Nanomechanical resonators can also find an application in the context of quantum computing. Coherent mechanical oscillators are suggested as coupling elements between phase qubits in a solid state quantum computer.[14] All the mentioned applications not only require the fabrication of a suitable oscillator but also a way to detect the motion of a nanomechanical resonator. Different schemes for detection have been proposed (see e.g. Ref. 1). The most promising candidates for sensitive readout are electrical devices, such as tunnel junctions or single electron transistors, incorporated on the same chip as the nanomechanical resonator.[1]

In the light of the possible applications it is necessary to obtain a theoretical understanding of nanomechanical resonators interacting with electrical devices on a chip. Theoretical descriptions of charge dynamics influenced by an oscillator have, up to now, mainly used a master equation technique. Mozyrsky and Martin investigated the model, where the transmission coefficient of a tunnel junction depends on the position of a nearby harmonic oscillator, in the zero temperature limit. They found that the oscillator acquires an effective temperature proportional to the junction bias voltage and also find the influence of the oscillator on the junction current at zero temperature.[15] Clerk and Girvin calculated the noise induced by a harmonic oscillator in a tunnel junction for dc and ac bias at zero temperature using a Markovian master equation.[16] The authors, together with Khomitsky found the current in a tunnel junction influenced by an oscillator for arbitrary system parameters.[17] The current and the noise power spectrum for an asymmetric junction in the high voltage limit were also calculated. Smirnov, Mourokh and Horing[18] analyzed the position fluctuations in the stationary state of a nanomechanical oscillator coupled to a tunnel junction for an exponential dependence of the tunneling amplitude on the oscillator position as well as the current through the tunnel junction. The theoretical descriptions of a nanomechanical oscillator interacting with a single electron transistor concentrated on the effects the single electron transistor, acting as a non-equilibrium environment, has on the oscillator. Rodrigues and Armour[19] derived a master equation for an oscillator-single electron transistor system and investigated stationary state properties and dynamics. Blencowe, Imbers and Armour[20] as well as Clerk and Bennett[21] considered the interaction of a superconducting single electron transistor with a nanomechanical resonator and discovered that for a particular source-drain voltage the single electron transistor can cool the oscillator and investigated a regime where the damping constant becomes negative .

In this article we consider a tunnel junction coupled to a harmonic oscillator as a model for a nanomechanical resonator interacting with a detector and concentrate on the features of the noise power spectrum that the oscillator induces in the tunnel junction. We are interested in this model system for several reasons: The induced noise provides a means to determine the temperature of the oscillator. A shot noise thermometer, utilizing a tunnel junction, was demonstrated by L. Spietz *et al.*[22] and was shown to work over a temperature range from 50 mK to 25 K. The noise induced by a nanomechanical oscillator in a SET has been successfully used to detect the temperature of a nanomechanical oscillator in two recent experiments,[7,8] in the region where the occupation number of the oscillator was large.

We also want to investigate if there exists a signature of the oscillator in the noise power spectrum even if the oscillator is in its ground state and the voltage is insufficient to excite the oscillator. A recent treatment of a similar system, considering a spin instead of an oscillator, claims a non-vanishing contribution to the noise power under these conditions.[23] A similar prediction could be made by blithely extending the result obtained by a Markovian master equation (e.g. from Ref. 17) into the region where the bias voltage is smaller than the oscillator frequency. Revisiting the junction/oscillator system utilizing a different technique will give us opportunity to investigate the question of the seemingly non-vanishing noise.

Also there has been a recent theoretical discussion about which current-current correlator is detected in a noise experiment. Lesovik and Loosen,[24] as well as Gavish *et al.*[25] argue, that a passive detector, e.g. a LC oscillator at zero temperature, can only detect the positive frequency part of the Fourier transform of the unsymmetrized current-current correlator. The oscillator coupled to a tunnel junction gives us the opportunity to revisit this question in the context of a more complicated system than a mere tunnel junction.

In this paper we will apply a Green's function technique to calculate the noise power spectrum of a tunnel junction coupled to an oscillator in the approximation of weak coupling, but for otherwise arbitrary parameters. The article is structured as follows: In section II we introduce the model Hamiltonian, and in section III we consider the stationary state of the oscillator using a Green's function technique. In section IV we calculate the average current through the junction as well as the unsymmetrized noise power spectrum. We consider application of the results in section V, discussing noise thermometry. We present the conclusions in section VI. Details of the calculations are presented in appendices.

II. OSCILLATOR INTERACTING WITH A TUNNEL JUNCTION

Let us consider the situation of a nanomechanical resonator, modelled as harmonic oscillator, interacting with a measuring device, modelled as a tunnel junction. The oscillator modulates the transmission amplitude of the junction thus changing the current and noise characteristics of the junction. The biased junction in turn acts a non-equilibrium environment for the oscillator, driving the oscillator from its initial state into a stationary thermal equilibrium state, albeit with a temperature different from the environment temperature of the tunnel junction. The Hamiltonian of the model system is

$$\hat{H} = \hat{H}_0 + H_l + H_r + \hat{H}_T \tag{2.1}$$

where $\hat{H}_0$ is the Hamiltonian for the isolated harmonic oscillator with bare frequency Ω_B and mass m. A hat marks operators acting on the oscillator degree of freedom. The Hamiltonians $H_{l,r}$ specify the isolated left and right electrodes of the junction

$$H_l = \sum_{\mathbf{l}} \varepsilon_{\mathbf{l}}\, c_{\mathbf{l}}^\dagger c_{\mathbf{l}} \quad , \quad H_r = \sum_{\mathbf{r}} \varepsilon_{\mathbf{r}}\, c_{\mathbf{r}}^\dagger c_{\mathbf{r}} \tag{2.2}$$

where $\mathbf{l},\mathbf{r}$ labels the quantum numbers of the single particle energy eigenstates in the left and right electrodes, respectively, with corresponding energies $\varepsilon_{\mathbf{l},\mathbf{r}}$ and annihilation and creation operators. The operator $\hat{H}_T$ describes the tunnelling,

$$\hat{H}_T = \hat{\mathcal{T}} + \hat{\mathcal{T}}^\dagger \quad , \quad \hat{\mathcal{T}} = \sum_{\mathbf{l},\mathbf{r}} \hat{T}_{\mathbf{lr}} c_{\mathbf{l}}^\dagger c_{\mathbf{r}} \tag{2.3}$$

with the tunneling amplitudes, $\hat{T}_{\mathbf{lr}} = \hat{T}_{\mathbf{rl}}^\dagger$, depending on the oscillator degree of freedom. Due to the interaction of the tunnel junction and the oscillator, the tunnelling amplitudes and thereby the conductance of the tunnel junction depend on the state of the oscillator. In the following we assume linear coupling between the oscillator position and the tunnel junction

$$\hat{T}_{\mathbf{lr}} = v_{\mathbf{lr}} + w_{\mathbf{lr}} \hat{x} \tag{2.4}$$

where $v_{\mathbf{lr}} = v_{\mathbf{rl}}^*$ is the unperturbed tunneling amplitude and $w_{\mathbf{lr}} = w_{\mathbf{rl}}^*$ its derivative with respect to the position of the oscillator.

To discuss the current and noise in the tunnel junction, the current operator is needed

$$\hat{I} = ei\left(\hat{\mathcal{T}} - \hat{\mathcal{T}}^\dagger\right). \tag{2.5}$$

The tunneling Hamiltonian consists of a part independent of the state of the oscillator and a part that depends on the state of the oscillator, so that the tunneling Hamiltonian, Eq. (2.3), can be presented on the form

$$\hat{H}_T = h_v + \hat{x} h_w. \tag{2.6}$$

For notational convenience, we have introduced the symbolic notation

$$h_u = \mathcal{T}_u + \mathcal{T}_u^\dagger, \quad \mathcal{T}_u = \sum_{\mathbf{lr}} u_{\mathbf{lr}} c_{\mathbf{l}}^\dagger c_{\mathbf{r}}, \quad u = v, w\,, \tag{2.7}$$

where the symbol $u_{\mathbf{lr}}$ can take the values $v_{\mathbf{lr}}$ or $w_{\mathbf{lr}}$. Similarly we can write the current operator, Eq. (2.5), as

$$\hat{I} = j_v + \hat{x} j_w, \tag{2.8}$$

with

$$j_u = i\left(\mathcal{T}_u - \mathcal{T}_u^\dagger\right), \quad u = v, w\,. \tag{2.9}$$

When calculating current and noise in the tunnel junction the following combinations of the model parameters $v_{\mathbf{lr}}$ and $w_{\mathbf{lr}}$ appear

$$\left\{ \begin{array}{c} G_{vv} \\ G_{ww} \\ G_{vw} \end{array} \right\} = 2\pi \sum_{\mathbf{lr}} \left\{ \begin{array}{c} v_{\mathbf{lr}}^2 \\ w_{\mathbf{lr}}^2 \\ v_{\mathbf{lr}} w_{\mathbf{lr}} \end{array} \right\} \left(-\frac{\partial f(\varepsilon_{\mathbf{l}})}{\partial \varepsilon_{\mathbf{l}}}\right) \delta(\varepsilon_{\mathbf{l}} - \varepsilon_{\mathbf{r}}), \tag{2.10}$$

where f is the Fermi function. Here and in the following, the transmission matrix elements are assumed real.

III. STATIONARY STATE PROPERTIES OF THE OSCILLATOR

In this section we consider the properties of the stationary state a harmonic oscillator reaches due to interaction with a tunnel junction. In the Keldysh technique (for a review see e.g. Ref. 26), we introduce the contour ordered oscillator matrix Green's function

$$D(\tau, \tau') = -i \left\langle \mathrm{T}_c \left(\hat{x}_H(\tau) \hat{x}_H(\tau')\right)\right\rangle. \tag{3.1}$$

The subscript H refers to an operator in the Heisenberg picture. Each of the times τ and τ' belong to one of the two branches of the Keldysh contour from $-\infty$ to $+\infty$. We will use the symbol τ to denote times on the contour, whereas t denotes real times. The branch index makes D a 2×2 matrix. The Keldysh matrix defined by Eq. (3.1) can be linearly transformed to the "triangular" form:

$$D = \begin{pmatrix} D^R & D^K \\ 0 & D^A \end{pmatrix}, \tag{3.2}$$

where D^R, D^A, and D^K are the retarded, advanced and Keldysh Green's functions, respectively.

For a stationary state, the elements of the Keldysh matrix are functions of only τ, τ', and the Fourier transformed oscillator Green's function satisfies the matrix Dyson equation

$$\left(D_0^{-1}(\omega) - \Pi(\omega)\right) D(\omega) = \hat{1} \tag{3.3}$$

where $D_0^{-1}(\omega) = \left[m(\omega^2 - \Omega_B^2)\right]$, Ω_B being the bare oscillator frequency, and the self-energy (polarization operator) is a matrix of the form

$$\Pi = \begin{pmatrix} \Pi^R & \Pi^K \\ 0 & \Pi^A \end{pmatrix}. \tag{3.4}$$

Assuming weak interaction of the oscillator with the tunnel junction, the self-energy can be taken to lowest order. Calculations, details of which can be found in appendix A, give the following expression for the polarization operator,

$$\Pi(\omega) = -iG_{ww} \begin{pmatrix} \omega & 2S_V(\omega) \\ 0 & -\omega \end{pmatrix} + R_{ww}^{(+)}(\omega)\hat{1}, \tag{3.5}$$

where the conductance G_{ww} is defined in Eq. (2.10) and the second term, the real part of the self-energy, is given by Eq. (A13); for ω of the order of the oscillator frequency, $R_{ww}^{(+)}(\omega)$ can be replaced by a constant $R_{ww}^{(+)}(\omega) \approx R_{ww}^{(+)}(0)$.[32] The function

$$S_V(\omega) = \frac{V+\omega}{2} \coth\frac{V+\omega}{2T} + \frac{V-\omega}{2} \coth\frac{V-\omega}{2T}, \tag{3.6}$$

where T is the temperature of the junction and $V = eU$, U being the applied dc-voltage, is proportional to the well-known value of the power spectrum of current noise of the isolated junction, see e.g. Ref. 27. Solving the Dyson equation, Eq. (3.3), the retarded and advanced oscillator Green's functions become

$$D^R(\omega) = m^{-1} \frac{1}{(\omega + i\gamma_e)^2 - \Omega^2}, \quad D^A(\omega) = \left(D^R(\omega)\right)^*, \tag{3.7}$$

where $\gamma_e = -\Im \Pi^R(\omega)/2m\omega$, the damping coefficient due to the coupling to the junction is

$$\gamma_e = \frac{G_{ww}}{2m}, \tag{3.8}$$

and the renormalized oscillator frequency is

$$\Omega^2 = \Omega_B^2 - \gamma_e^2 + \frac{1}{m} R_{ww}^{(+)}(0). \tag{3.9}$$

For the Keldysh component we obtain

$$D^K(\omega) = \left(D^R(\omega) - D^A(\omega)\right) \frac{S_V(\omega)}{\omega}. \tag{3.10}$$

Additionally to the environment provided by the coupling to the tunnel junction a nanomechanical oscillator is also subject to an intrinsic environment, e.g. phonons, acting as a heat bath and leading to damping. This additional heat bath, which we take to have the same temperature, T, as the junction, can be added phenomenologically, or explicitly by adding the interaction with a bath of harmonic oscillators (as introduced in Refs. 16,17). The total damping coefficient for the harmonic oscillator will then be the sum of the damping coefficients stemming from the tunnel junction, γ_e, and the heat bath,

γ_0, giving a total damping coefficient $\gamma = \gamma_e + \gamma_0$. As a consequence of the additional heat bath the relation between the oscillator Green's functions, Eq. (3.10), is modified according to

$$D^K(\omega) = \left(D^R(\omega) - D^A(\omega)\right)\left(\frac{\gamma_e}{\gamma}\frac{S_V(\omega)}{\omega} + \frac{\gamma_0}{\gamma}\coth\frac{\omega}{2T}\right), \tag{3.11}$$

with the damping coefficient γ_e given in Eq. (3.7). The relation for the Keldysh Green's function, Eq. (3.11), is characteristic of the oscillator interacting with the environment which is in a non-equilibrium but steady state, and only in the absence of a bias voltage reduces it to the fluctuation-dissipation relation for an oscillator in thermal equilibrium at temperature T. However, since the coupling of the oscillator to the junction is weak, $\gamma \ll \max(V, \Omega, T)$, the oscillator spectral function is peaked at its frequency Ω, and Eq. (3.11) can be written in the standard form

$$D^K(\omega) = \left(D^R(\omega) - D^A(\omega)\right)\coth\frac{\omega}{2T^*}, \tag{3.12}$$

where the temperature T^* characterizing the stationary state of the oscillator is given by

$$\coth\frac{\Omega}{2T^*} = \left(\frac{\gamma_e}{\gamma}\frac{S_V(\Omega)}{\Omega} + \frac{\gamma_0}{\gamma}\coth\frac{\Omega}{2T}\right). \tag{3.13}$$

The temperature of the oscillator, T^*, depends not only on the environment temperature T, but also on the bias voltage of the junction V, as well as the relative coupling strengths γ_e/γ and γ_0/γ. For zero bias voltage the effective temperature reduces to the environment temperature. A frequency dependent effective temperature in the context of nanomechanical systems was also discussed by Clerk,[28] as well as by Clerk and Bennett,[21] for coupling to a general environment. Clerk derived a Langevin equation for a harmonic oscillator coupled to a fluctuating force, and obtained a relation similar to Eq. (3.13), defining a frequency dependent effective temperature.

IV. NOISE PROPERTIES OF THE JUNCTION

In this section we shall study how a harmonic oscillator interacting with a tunnel junction influences the current noise in the tunnel junction in the steady state. The noise properties of the junction current can be obtained in perturbation theory. The noise spectrum is specified by the current-current correlation function

$$\left\langle \delta\hat{I}_H(t)\delta\hat{I}_H(t')\right\rangle = \left\langle \hat{I}_H(t)\hat{I}_H(t')\right\rangle - I^2, \tag{4.1}$$

where the current operator is given by Eq. (2.8) and the subscript H refers to an operator in the Heisenberg picture. For calculational convenience we introduce the current-current correlator with the time arguments lying on the Keldysh contour

$$S(\tau,\tau') = \left\langle \mathrm{T}_c\left(\delta\hat{I}_H(\tau)\delta\hat{I}_H(\tau')\right)\right\rangle. \tag{4.2}$$

It can be written in the interaction picture as

$$S(\tau,\tau') = \left\langle \mathrm{T}_c\left(e^{-i\int_c d\tau \hat{H}_T(\tau)}\hat{I}(\tau)\hat{I}(\tau')\right)\right\rangle - I^2, \tag{4.3}$$

where c denotes the Keldysh contour. To obtain the noise spectrum from the current-current correlator Eq. (4.3) we introduce Keldysh indices $i, j = 1, 2$, that label the contour, e.g., $i = 1$ for the forward contour or $i = 2$ for the backward contour, and revert to using the real times t and t', so that

$$S(\tau,\tau') \to S^{ij}(t-t'). \tag{4.4}$$

Finally, taking the Fourier transform of Eq. (4.4) we obtain

$$S^{ij}(\omega) = \int_{-\infty}^{\infty} dt\, e^{i\omega t} S^{ij}(t). \tag{4.5}$$

For calculational purposes it is sufficient to consider the real-time unsymmetrized current-current correlator

$$S^<(t-t') = S^{12}(t-t') = \langle I(t)I(t')\rangle\,, \tag{4.6}$$

since all other correlators can be derived from it, e.g. $S^>(t-t') = \langle I(t')I(t)\rangle = S^<(t'-t)$.

A. Current

First we calculate the average current I to second order in the tunneling amplitude

$$I(t) = -i\int_c d\tau \left\langle \mathrm{T}_c\left(\hat{H}_T(\tau)\hat{I}(t)\right)\right\rangle. \tag{4.7}$$

Inserting the tunneling Hamiltonian Eq. (2.6) and the current operator Eq. (2.8) we get two contributions to the current

$$I(t) = I_{vv}(t) + I_{ww}(t), \tag{4.8}$$

where

$$I_{vv}(t) = i\int_c d\tau \Pi^{(-)}_{vv}(t-\tau), \tag{4.9}$$

is specified by the function $\Pi^{(-)}$ defined in Eq. (A1), and the contribution due to the coupling to the oscillator is

$$I_{ww}(t) = -\int_c d\tau D(t-\tau)\Pi^{(-)}_{ww}(t-\tau). \tag{4.10}$$

Here we have replaced the free oscillator Green's function appearing to lowest order in the coupling with the full oscillator Green's function, i.e., resumming the terms due to the interaction in order to take into account the effect of the interaction with the tunnel junction on the oscillator. The oscillator Green's function was obtained in section III, and in the following this substitution will be performed immediately in order to properly account for the properties of the oscillator.

The steady state current for an isolated junction becomes

$$I_{vv} = G_{vv} V, \tag{4.11}$$

in accordance with the retarded part of $\Pi_{vv}^{(-)}$, Eq. (A12) and numerous previous studies.

Similarly we get, according to Eq. (4.10), for the contribution induced by the coupling to the oscillator

$$I_{ww} = -\frac{i}{2} G_{ww} \int_{-\infty}^{\infty} d\omega J(\omega). \tag{4.12}$$

where

$$J(\omega) = V D^K(\omega) - \left[D^R(\omega) - D^A(\omega)\right] \Delta_V(\omega), \tag{4.13}$$

with the oscillator Green's functions found in section III, and

$$\Delta_V(\omega) = \frac{V+\omega}{2} \coth\frac{V+\omega}{2T} - \frac{V-\omega}{2} \coth\frac{V-\omega}{2T}. \tag{4.14}$$

Since the oscillator Green's functions are peaked at the oscillator frequency the integral in Eq. (4.12) can be evaluated and using the expression for the Keldysh Green's functions, Eq. (3.12), we get

$$I_{ww} = \frac{1}{2} \tilde{G}_{ww} \left[\mathcal{V}_+ N^* + \mathcal{V}_- \left(N^* + 1\right)\right], \tag{4.15}$$

where

$$\tilde{G}_{ww} = \frac{\hbar}{m\Omega} G_{ww} \tag{4.16}$$

and we introduced the short notation

$$\mathcal{V}_\pm = V \pm \Delta_V(\Omega) \tag{4.17}$$

and the occupation number of the oscillator is given by the Bose function

$$N^* = \frac{1}{e^{\Omega/T^*} - 1}, \tag{4.18}$$

where the temperature of the oscillator, T^*, is specified in Eq. (3.13). From Eq. (4.15) it can be seen that two oscillator-assisted tunnelling processes contribute to the tunneling electrons gaining energy from the oscillator and loosing energy to the oscillator, respectively. The combinations $\mathcal{V}_\pm$ give the phase space volume for these processes.

B. Current-current correlator

Next we turn to calculate the current-current correlator. For an isolated junction, the noise is given by the second order expression in the tunneling amplitude since the fourth order correction only introduces a small featureless correction. However, when the junction is coupled to a quantum system exhibiting resonant behavior, as in the case of the oscillator, the interaction of the system with the tunnel junction can markedly increase the fourth order noise correction at the resonance and combinational frequencies. We will be interested in the resonant contributions and will therefore consider the current-current correlator to fourth order.

Expanding the expression for the current-current correlator, Eq. (4.3), to fourth order in the tunneling amplitude we obtain

$$S(\tau,\tau') = S_{(2)}(\tau,\tau') + S_{(4)}(\tau,\tau'), \tag{4.19}$$

with the correlator to second order in the tunneling amplitude

$$S_{(2)}(\tau,\tau') = \left\langle \mathrm{T}_c \left(\hat{I}(\tau)\hat{I}(\tau')\right)\right\rangle \tag{4.20}$$

and the correlator to fourth order in the tunneling amplitude

$$S_{(4)}(\tau,\tau') = \\ -\frac{1}{2}\int_c d\tau_1 \int_c d\tau_2 \left\langle \mathrm{T}_c \left(\hat{H}_T(\tau_1)\hat{H}_T(\tau_2)\hat{I}(\tau)\hat{I}(\tau')\right)\right\rangle_L, \tag{4.21}$$

where the subscript L denotes that only linked diagrams contribute to the expression, since the disconnected diagrams are cancelled by the current squared term. We first investigate the second order in tunneling contribution to the current-current correlator, resulting in the noise floor.

1. Noise floor

In this section we are going to discuss the noise floor. The main contribution to the floor comes from the second order contribution to the noise spectrum.

To second order in the tunneling amplitudes

$$S_{(2)}(\tau,\tau') = S_{vv}(\tau,\tau') + S_{ww}(\tau,\tau'), \tag{4.22}$$

consists of the correlator for the isolated junction

$$S_{vv}(\tau,\tau') = \langle \mathrm{T}_c\left(j_v(\tau)j_v(\tau')\right)\rangle = -i\Pi_{vv}^{(+)}(\tau,\tau') \tag{4.23}$$

and a contribution induced by the coupling to the oscillator

$$S_{ww}(\tau,\tau') = \langle \mathrm{T}_c\left(j_w(\tau)j_w(\tau')\right)\rangle \langle \mathrm{T}_c\left(\hat{x}(\tau)\hat{x}(\tau')\right)\rangle, \tag{4.24}$$

and therefore

$$S_{ww}(\tau,\tau') = -\Pi^{(+)}_{ww}(\tau,\tau')D(\tau,\tau'). \quad (4.25)$$

The correlator $S^{<}_{vv}$ becomes in terms of the bubbles $\Pi^{R,K,A}$

$$S^{<}_{vv}(t) = -\frac{i}{2}\left[\Pi^{K}_{(+)}(t) - \Pi^{R}_{(+)}(t) + \Pi^{A}_{(+)}(t)\right], \quad (4.26)$$

which are specified in accordance with Eq. (A10). We get the noise spectrum of an isolated junction

$$S^{<}_{vv}(\omega) = G_{vv}\left[S_V(\omega) - \omega\right]. \quad (4.27)$$

This result has been previously obtained by Aguado and Kouwenhoven for the quantum point contact.[29]

The symmetrized noise spectrum of an isolated junction

$$S^{K}_{vv}(\omega) = S^{<}_{vv}(\omega) + S^{>}_{vv}(\omega), \quad (4.28)$$

becomes the well known result[30]

$$S^{K}_{vv}(\omega) = 2G_{vv}S_V(\omega), \quad (4.29)$$

where we used the property, $S^{<}(\omega) = S^{>}(-\omega)$. In the limit of zero voltage the noise floor reduces to Johnson-Nyquist noise. The Markovian master equation approach employed in Ref. 16 and Ref. 17 is not able to reproduce the correct frequency dependence, but only captures the zero frequency noise, and gives a constant noise floor with the magnitude of $S^{K}(0)$.

At low temperatures $T \ll V$, the unsymmetrized noise spectrum $S^{<}_{vv}$ is controlled by the voltage V: at positive frequencies $\omega > |V|$ the noise is exponentially small, $S^{<}_{vv}(\omega) \approx 0$, and $S^{<}_{vv}$ increases linearly with the distance from the threshold $\omega = |V|$:

$$S^{<}_{vv}(\omega) = G_{vv}\left[(V-\omega)\theta(V-\omega) + (-V-\omega)\theta(-V-\omega)\right]. \quad (4.30)$$

We see that at $T = 0$, the noise power $S^{>}_{vv}(\omega)$ is proportional to the phase space volume available for tunnelling events, i. e., the total number of electron-hole states, with an electron on one side of the junction and a hole on the other side, with the excitation energy ω. Obviously, this result holds only in the frequency range where the electron density of states can be considered as a constant.

For the contribution to the noise induced by the oscillator, Eq. (4.25), we obtain

$$S^{<}_{ww}(\omega) = 2G_{ww}\int_{-\infty}^{\infty} d\omega_1\left[S_V(\omega_1) - \omega_1\right]D^{<}(\omega-\omega_1) \quad (4.31)$$

where $D^{<}(\omega) = [D^{K}(\omega) - D^{R}(\omega) + D^{A}(\omega)]/2$. Recalling that the oscillator Green's functions are peaked at the oscillator frequency, the resulting contribution to the noise can be presented as

$$\begin{aligned} S^{<}_{ww}(\omega) &= \frac{1}{2}\tilde{G}_{ww}\left[N^{*}\left(S_V(\omega_-) - \omega_-\right)\right. \\ &= \left. + \left(N^{*}+1\right)\left(S_V(\omega_+) - \omega_+\right)\right] \quad (4.32) \end{aligned}$$

where $\omega_{\pm} = \omega \pm \Omega$ and the conductance $\tilde{G}_{ww}$ is given by Eq. (4.16).

The second order contribution of the oscillator to the noise is similar to that of an isolated junction: with a different coupling, $\tilde{G}_{ww}$ in the place of G_{vv}, the noise is given by the same expressions apart from the frequency shift $\pm\Omega$. The two terms in Eq. (4.32) give the noise contribution due to processes of electron tunneling accompanied by absorption or emission of oscillator quanta, respectively. At low temperatures $T \ll \Omega, V$, when the oscillator approaches the ground state, $N^{*} \to 0$, the noise $S^{<}_{ww}(\omega)$ is seen to vanish for frequencies larger than $V-\Omega$.

As expected, there will be no contribution to the noise Eq. (4.32) for positive frequencies at zero temperature and voltage $|V| < \Omega$, , since the voltage is insufficient to excite the oscillator.

To summarize, the second order contribution to the noise spectrum consists of the noise spectrum of an isolated junction, and a part that depends on the state of the oscillator. The second order contribution to the noise spectrum is frequency dependent and changes on the scale of $\max(V, \Omega, T)$.

2. Resonant contribution to the current-current correlator

In this section we consider the resonant contribution to the noise spectrum, that is, sharp peaks in the noise power with a width given by the oscillator damping γ. Technically, the resonant contribution originates from the fourth order terms Eq. (4.21). Among various Feynman diagrams generated by the expression in Eq. (4.21), we are therefore interested only in those that produce resonant features. The diagrams of interest are shown in Figs. 1, 2, where the wiggly lines represent oscillator Green's functions, while the bubbles labelled with (–) are antisymmetric combinations of bubbles of electron Green's functions as specified in Fig. 3.

The diagrams with a single oscillator line in Fig. 1 generate sharp features in the noise spectrum at $\omega = \pm\Omega$, while the two-line diagrams in Fig. 2 are responsible for noise peaks in the vicinity of $\omega = 0$ and $\omega = \pm 2\Omega$.

An example of a 4-th order diagram whose contribution is a featureless function of frequency ω is shown in Fig. 4. In this diagram, the frequency of the oscillator line is integrated over and therefore the resonant contribution is absent. Being small compared with the second order contribution to noise, this diagrams and other diagrams of this type can be discarded.

We will denote the resonant contribution of the fourth order diagrams to the current correlator by $S^{<}_{res}$. It is given by the sum

$$S^{<}_{res}(\omega) = S^{<}_{v^2w^2}(\omega) + S^{<}_{w^4}(\omega), \quad (4.33)$$

where $S^{<}_{v^2w^2}(\omega)$ and $S^{<}_{w^4}(\omega)$ are the contributions of the diagrams in Fig. 1 and Fig. 2, respectively.

Evaluating the expression represented in Fig. 1 by performing the summation over the contour indices k and l,

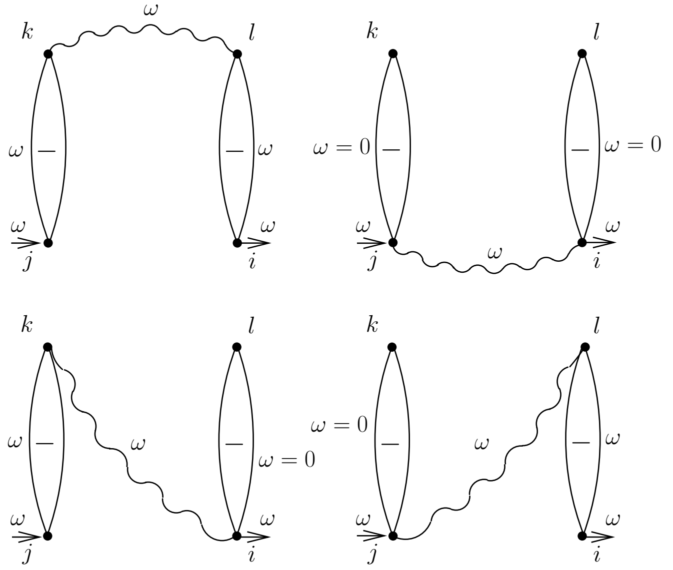

Figure 1: The four diagrams proportional to v^2w^2 that contribute to the peak at the oscillator frequency. The wiggly lines represent the oscillator propagator and the bubbles are defined according to Fig. 3. The short arrows labelled by ω indicate the external frequency, i, j, k and l are Keldysh indices.

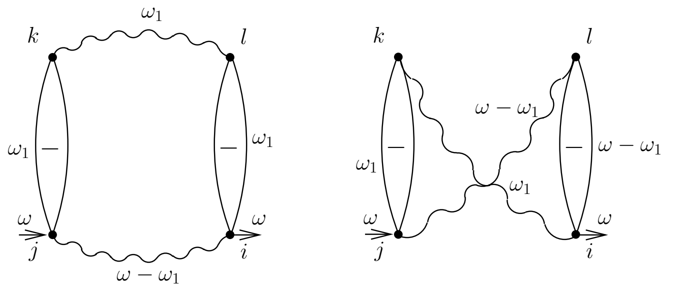

Figure 2: The two diagrams proportional w^4 that contribute to the peaks at $\omega = 0$ and $\omega = 2\Omega$. The wiggly lines represent the oscillator propagator and the bubbles are defined according to Fig. 3. The short arrows labelled by ω represent the external frequency, the indices i, j, k and l are Keldysh indices. Integration over ω_1 is implicit.

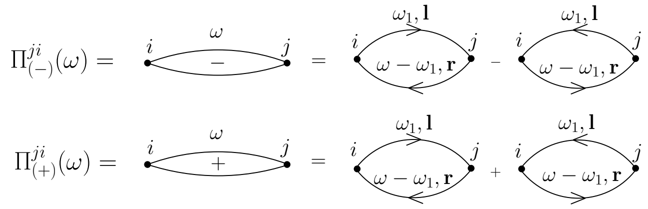

Figure 3: The diagrammatic representation of the junction Green's functions, Eq. (A1). Each bubble represents a linear combination of two electron bubble diagrams, with the Keldysh indices i and j. Integration over the internal frequency, ω_1, is implicit.

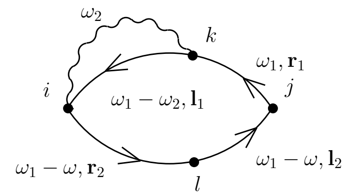

Figure 4: A diagram representing a non-bubble contribution to the noise spectrum. The solid lines represent left and right side electron Green's function, the wiggly line represents the oscillator Green's functions. Integration over ω_1 and ω_2 is implied.

and inserting the bubbles and Green's functions we get (see Appendix B for detail)

$$\tilde{S}^{<}_{v^2w^2}(\omega) = -2iG^2_{vw}VJ(\omega), \tag{4.34}$$

where the function $J(\omega)$ is specified in Eq. (4.13) and the conductance given in Eq. (2.10). This contribution to the noise spectrum is peaked at $\omega = \pm\Omega$ due to the resonant behavior of the harmonic oscillator spectral function. The dependence of the peak height on the system parameters will be discussed in section IV C.

The contribution containing two oscillator Green's functions in Fig. 2 is calculated in Appendix B to give

$$\begin{aligned}\tilde{S}^{<}_{w^4}(\omega) &= \frac{G^2_{ww}}{2}\int_{-\infty}^{\infty} d\omega_1 J(\omega_1)J(\omega-\omega_1)\\ &+ \frac{G^2_{ww}}{2}\int_{-\infty}^{\infty} d\omega_1 A(\omega_1)A(\omega-\omega_1)\\ &\times \left[V-\Delta_V(\omega_1)\right]\left[V-\Delta_V(\omega-\omega_1)\right]\end{aligned} \tag{4.35}$$

where we introduced the oscillator spectral function $A(\omega) = i\left[D^R(\omega) - D^A(\omega)\right]$. The dependence of the contribution to the noise spectrum, Eq. (4.35), on system parameters will be discussed in section IV C.

C. Resonant contributions to the noise spectrum

The resonant contribution to the noise consists of peaks at zero frequency, $\omega = 0$, the oscillator frequency, $\omega = \pm\Omega$, and twice the oscillator frequency, $\omega = \pm 2\Omega$. Their heights depend inversely on the damping coefficient of the oscillator, γ, and their width is proportional to the damping coefficient. In this section we will discuss the properties of the peaked structure and its dependence on environment parameters like bias voltage and temperature. The peaked contribution to the noise can be written as

$$S^<_p(\omega) = S^<_{-2}(\omega) + S^<_{-1}(\omega) + S^<_0(\omega) + S^<_1(\omega) + S^<_2(\omega), \tag{4.36}$$

where $S^<_0(\omega)$ describes the peaked contribution to the noise spectrum at zero frequency, $S^<_{\pm 1}(\omega)$ the contributions at the oscillator frequency and $S^<_{\pm 2}(\omega)$ the contributions at twice the oscillator frequency.

The peaks at the frequencies $\omega = \pm\Omega$ originate from the bubble contribution, $\tilde{S}^{<}_{v^2w^2}(\omega)$, given in Eq. (4.34). Inserting the explicit form of the oscillator Green's functions, Eqs. (3.7, 3.12), in Eq. (4.34), and using that the oscillator Green's functions are peaked at the oscillator frequency, we obtain for the resonant contribution to the noise spectrum at the oscillator frequency

$$S^{<}_{\pm 1}(\omega) = \frac{\gamma^2}{(\omega \mp \Omega)^2 + \gamma^2} P_\Omega. \tag{4.37}$$

The peak height at the resonance frequency is

$$P_\Omega = \frac{1}{\gamma}\tilde{G}^2_{vw} V \left[N^* \mathcal{V}_+ + (N^*+1)\mathcal{V}_-\right], \tag{4.38}$$

where $\mathcal{V}_\pm$ is given by Eq. (4.17) and N^* is the occupation number of the oscillator. The peak height scales inversely with the damping coefficient γ. For large voltages $V \gg \Omega$ the peak height is linear in the occupation number N^* and quadratic in the voltage. The result, Eq. (4.38), extends the result from the Markovian master equation calculation in Refs. 16,17 to voltages smaller than the oscillator frequency.

The peaks at frequencies $\omega = 0$ and $\omega = \pm 2\Omega$ come from the fourth order contribution quadratic in the oscillator Green's functions, Eq. (4.35). The remaining integration in Eq. (4.35) can be done and the dominating contribution for weak damping comes from the poles of the oscillator Green's functions. Collecting the contributions to the peak at zero frequency we get

$$S^{<}_0(\omega) = \frac{4\gamma^2}{4\omega^2 + 4\gamma^2} P_0 \tag{4.39}$$

where the peak height P_0 is

$$P_0 = \frac{1}{2\gamma}\tilde{G}^2_{ww} V \left[N^{*2}\mathcal{V}_+ + (N^*+1)^2\mathcal{V}_-\right]. \tag{4.40}$$

The peak height at zero frequency also scales inversely with the damping coefficient γ and depends quadratically on the conductance $\tilde{G}_{ww}$. The result for the peak at zero frequency, Eq. (4.39), coincides with the result obtained by using a Markovian master equation approach as in Ref. 17, which therefore correctly captures the low frequency noise.

The contributions at double the oscillator frequency differ for positive and negative frequency. In the vicinity of $\omega = 2\Omega$,

$$S^{<}_2(\omega) = \frac{4\gamma^2}{(\omega - 2\Omega)^2 + 4\gamma^2} P_{2\Omega} \tag{4.41}$$

with the peak height given by

$$P_{2\Omega} = \tilde{G}^2_{ww}\frac{1}{4\gamma} V N^* \left[\mathcal{V}_+ N^* + \mathcal{V}_-(N^*+2)\right] \tag{4.42}$$

and for $\omega \sim -2\Omega$

$$S^{<}_{-2}(\omega) = \frac{4\gamma^2}{(\omega + 2\Omega)^2 + 4\gamma^2} P_{-2\Omega} \tag{4.43}$$

with the peak height

$$\begin{aligned} P_{-2\Omega} &= \tilde{G}^2_{ww}\frac{1}{4\gamma} V (N^*+1) \\ &\quad \times \left[\mathcal{V}_+(N^*-1) + \mathcal{V}_-(N^*+1)\right]. \end{aligned} \tag{4.44}$$

The peaks at double the oscillator frequency also show resonant behavior, i.e. the peak height increases with decreasing damping. For large voltages and occupation number, $N^* \gg 1$, it depends quadratically on the oscillator occupation number and the voltage.

The peak heights depend on the bias V, the environment temperature T and the relative coupling strengths γ_e/γ and γ_0/γ. In the following we discuss the peak heights in some limiting cases.

1. *Dominant coupling to the tunnel junction*

If the coupling between the oscillator and the tunnel junction is much stronger than the coupling to the thermal environment, $\gamma_e \gg \gamma_0$, the occupation number of the oscillator is, according to Eq. (3.13), given by

$$N^* = \frac{1}{2}\left(\frac{S_V(\Omega)}{\Omega} - 1\right). \tag{4.45}$$

a. For high temperatures $T \gg V, \Omega$ the occupation number of the oscillator becomes $N^* \approx T/\Omega$. The functions $\mathcal{V}_\pm = V$ in this limit. The peak heights of the resonant contributions to the noise spectrum are then at the oscillator frequency

$$P_\Omega = \tilde{G}^2_{vw}\frac{4}{\gamma} V^2 \frac{T}{\Omega}, \tag{4.46}$$

and is linear in the environment temperature. The peak height at zero frequency is

$$P_0 = \tilde{G}^2_{ww}\frac{1}{\gamma} V^2 \left(\frac{T}{\Omega}\right)^2, \tag{4.47}$$

and depends quadratically on the environment temperature, whereas the peak heights at twice the oscillator frequency become

$$P_{\pm 2\Omega} = \tilde{G}^2_{ww}\frac{1}{2\gamma} V^2 \left(\frac{T}{\Omega}\right)^2, \tag{4.48}$$

which also scales quadratically in the environment temperature. For dominant coupling to the junction $\gamma \propto G_{ww}$ and therefore all the peak heights becomes linear functions of the conductance. The peak heights for high voltages $V \gg T, \Omega$ can be obtained from the peak heights at high temperatures by replacing $T \to V/2$ in Eqs. (4.46, 4.47, 4.48).

b. At low temperatures, $T \ll \Omega$, and low voltages $V < \Omega$, the occupation number approaches zero, $N^* \approx 0$. The peaks at the oscillator frequency as well as the peak at zero frequency and the peak at $\omega = 2\Omega$ disappear

$$P_0 = P_\Omega = P_{2\Omega} = 0. \tag{4.49}$$

At $\omega = -2\Omega$ we obtain a dip in the noise spectrum with depth

$$P_{-2\Omega} = -\tilde{G}^2_{ww}\frac{1}{2\gamma}V^2.$$

Since in nanomechanical systems the coupling strength between the junction and the oscillator can be tuned, we will also discuss the case where the thermal environment dominates over the junction.

2. *Dominant coupling to the thermal environment*

Another limiting case to consider is the situation when the coupling to the thermal environment dominates over the coupling to the tunnel junction, $\gamma_0 \gg \gamma_e$. In the high temperature limit, $T \gg V, \Omega$, we obtain the same results for the peak heights as in the case of dominant coupling to the junction, since for both cases the junction and the thermal environment act as thermal equilibrium environments with temperature T. For high voltages and temperatures much larger than the oscillator frequency, $V \gg T \gg \Omega$, we get

$$N^* = \frac{\gamma_e}{\gamma}\frac{|V|}{2\Omega} + \frac{\gamma_0}{\gamma}\frac{T}{\Omega} \gg 1. \tag{4.50}$$

The peak height at the oscillator frequency is

$$P_\Omega = \tilde{G}^2_{vw}\frac{8}{\gamma}V^2\left(\frac{\gamma_e}{\gamma}\frac{|V|}{2\Omega} + \frac{\gamma_0}{\gamma}\frac{T}{\Omega}\right). \tag{4.51}$$

The peak height at zero frequency is

$$P_0 = \tilde{G}^2_{ww}\frac{1}{\gamma}V^2\left(\frac{\gamma_e}{\gamma}\frac{|V|}{2\Omega} + \frac{\gamma_0}{\gamma}\frac{T}{\Omega}\right)^2. \tag{4.52}$$

The peak height at twice the oscillator frequency

$$P_{\pm 2\Omega} = \tilde{G}^2_{ww}\frac{1}{2\gamma}V^2\left(\frac{\gamma_e}{\gamma}\frac{|V|}{2\Omega} + \frac{\gamma_0}{\gamma}\frac{T}{\Omega}\right)^2 \tag{4.53}$$

is also quadratic in the occupation number. All peak heights depend inversely on the damping coefficient γ, since in the parenthesis γ appears only in the relative coupling strengths γ_e/γ and γ_0/γ.

For low temperatures $T \ll V, \Omega$ we distinguish two regimes. For low voltages, $V < \Omega$, we obtain the same results as in the case for strong coupling to the junction since $N^* \approx 0$. For voltages $V \gg \Omega$

$$N^* = \frac{\gamma_e}{\gamma}\frac{|V|}{2\Omega}. \tag{4.54}$$

The functions $\mathcal{V}_\pm \approx V \pm \Omega$ and we obtain for the peak height at the oscillator frequency

$$P_\Omega = \tilde{G}^2_{vw}\frac{1}{\gamma}V^2\left(2\left(\frac{\gamma_e}{\gamma}\frac{|V|}{2\Omega}\right) + 1\right), \tag{4.55}$$

for the peak height at zero frequency

$$P_0 = \tilde{G}^2_{ww}\frac{1}{2\gamma}V^2\left[2\left(\frac{\gamma_e}{\gamma}\frac{V}{2\Omega}\right)^2 + 2\left(\frac{\gamma_e}{\gamma}\frac{|V|}{2\Omega}\right) + 1\right] \tag{4.56}$$

and the peak heights at twice the oscillator frequency

$$P_{2\Omega} = \tilde{G}^2_{ww}\frac{1}{2\gamma}V^2\left(\frac{\gamma_e}{\gamma}\frac{|V|}{2\Omega}\right)\left[\left(\frac{\gamma_e}{\gamma}\frac{|V|}{2\Omega}\right) + 1\right] \tag{4.57}$$

and

$$P_{-2\Omega} = \tilde{G}^2_{ww}\frac{1}{2\gamma}V\left[\left(\frac{\gamma_e}{\gamma}\frac{|V|}{2\Omega}\right) + 1\right]\left[V\left(\frac{\gamma_e}{\gamma}\frac{|V|}{2\Omega}\right) - \Omega\right]. \tag{4.58}$$

We have obtained the peak heights of the resonant peaks in the noise spectrum of a tunnel junction coupled to a harmonic oscillator for arbitrary parameters V, T and γ_e, γ_0. We find that if the oscillator approaches the ground state, $N^* \to 0$, the peaks at positive frequencies as well as the peak at zero vanish. In the next section we are going to discuss an application of the properties of the peaks in the noise spectrum of the junction: noise thermometry.

V. NOISE THERMOMETRY

In experiments trying to cool a nanomechanical oscillator to the ground state a diagnostic tool is needed to check the state of the oscillator and to confirm, eventually, that the oscillator really is in the ground state. Coupling the oscillator to an electrical device, e.g. a tunnel junction, gives us the means to determine the state of the oscillator, since the oscillator influences the current and noise in the junction. In a noise thermometry setup we want to use the noise that the oscillator induces in the tunnel junction to determine the temperature or occupation number of the oscillator. The noise spectrum of a tunnel junction weakly coupled to an oscillator consists, as demonstrated, of three peaks, one at zero frequency one at the oscillator frequency and one at twice the oscillator frequency. If the oscillator couples weakly to the junction the highest peak will be at the oscillator frequency. Let us therefore use the peak height, Eq. (4.38) to determine the oscillator temperature. Fig. 5 shows the dependence of the peak height at the oscillator frequency, Eq. (4.38), as a function of the environment temperature T for different bias voltages V and dominant coupling to the junction $\gamma_e/\gamma = 10$. The solid line shows the peak height for $V = 0.1\Omega$. It decreases monotonically for decreasing temperature, and becomes for small temperatures exponentially small. The

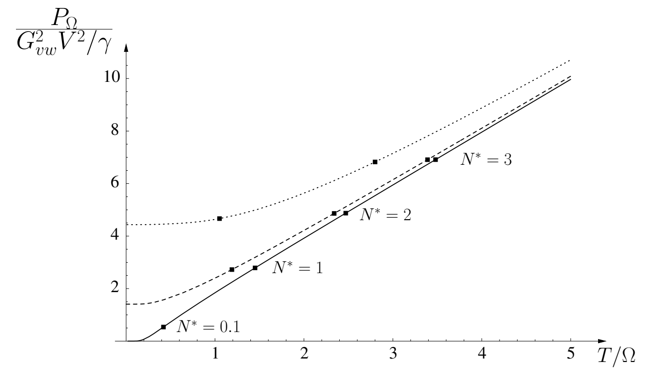

Figure 5: The peak height of the noise at the oscillator frequency, P_Ω, as a function of the environment temperature T for different bias voltages and dominant coupling to the junction, $\gamma_e/\gamma_0 = 10$, and $\tilde{G}_{vw} = 0.1$. The full line shows the temperature dependence of the peak height for $V = 0.1\Omega$, the dashed line for $V = 2\Omega$ and the dotted line for $V = 5\Omega$. As the environment temperature approaches zero, the peak height vanishes for voltages $V < \Omega$, but stays finite for voltages $V > \Omega$. The square dots mark different occupation numbers of the oscillator as indicated.

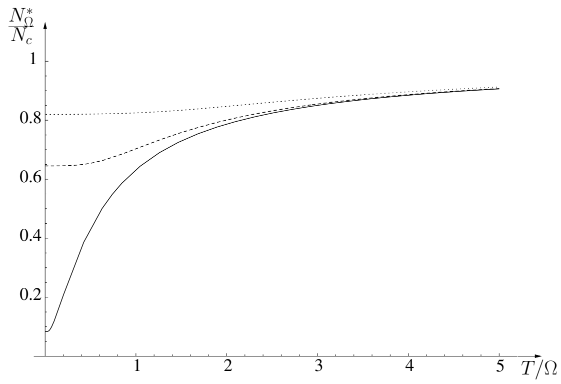

Figure 6: The relative occupation number of the oscillator N^*/N_c as a function of the environment temperature T for different bias voltages V and dominant coupling to the junction, $\gamma_e/\gamma_0 = 10$, with $\tilde{G}_{vw} = 0.1$. The full line shows the relative occupation number for low voltage, $V = 0.1\Omega$, the dashed line and the dotted line show the relative occupation number for high voltages, $V = 2\Omega$ and $V = 5\Omega$, respectively. For high temperatures $T \gg \Omega$, the relative occupation number approaches unity. For low temperatures the relative occupation number can depart considerably from unity to approach $1/(1 + \gamma\Omega/\gamma_e V)$.

dashed and the dotted line show the peak height for voltages larger than the oscillator frequency ($V = 2\Omega$ and $V = 5\Omega$ respectively). For voltages higher than the oscillator frequency the peak height stays finite even for zero environment temperature. Previous calculations of the noise spectrum, by Clerk and Girvin,[16] as well as by the authors with Khomitsky,[17] using a master equation approach, were only applicable for large voltages $V \gg \Omega$. We show here that the peak height at the oscillator frequency vanishes if the oscillator approaches the ground state.

For high occupation numbers (i.e. high temperatures or high voltage, $\max(V, T) \gg \Omega$), the peak height at the oscillator frequency is a good measure of the occupation number of the oscillator. In this classical region we can obtain the oscillator occupation number by $N^* \approx N_c$, where the dimensionless peak height is given by

$$N_c = \gamma \frac{P_\Omega}{2\tilde{G}_{vw}^2 V^2}. \tag{5.1}$$

In the region of low occupation numbers we obtain the relation for the occupation number

$$N^* = N_c - \frac{1}{2}\frac{\mathcal{V}_-}{V}. \tag{5.2}$$

We note that for low temperatures and voltages the oscillator occupation number is not simply proportional to the peak height at the oscillator frequency. The relative occupation number N^*/N_c is shown in Fig. 6 for dominant coupling to the junction and different bias voltages. At high temperatures the relative occupation number approaches unity, for low temperatures and low voltages it departs from unity.

The calculation of the noise spectrum presented in this paper enables us to extract the occupation number of the oscillator for arbitrary bias voltages and temperatures and gives a tool to interpret data from noise thermometry experiments also in the range of low occupation numbers, that could not be described consistently earlier.

VI. CONCLUSION

We have considered a nanomechanical resonator interacting with a dc-biased tunnel junction. We model the resonator as a harmonic oscillator in a model where the tunneling amplitude is modulated by coupling to the harmonic oscillator position. Employing a Green's function technique we calculated the properties of the stationary state of the oscillator, obtaining that the coupling to the junction introduces damping and heats the oscillator. The expression Eq. (3.13), for the temperature of a harmonic oscillator coupled to a tunnel junction was also obtained in Ref. 17 using a Markovian master equation approach. The reason for the coinciding results derived by two different techniques is due to the same weak coupling approximation made in both derivations. Both, the perturbative Green's function calculation presented here and the Markovian master equation can only be applied for weak coupling between the environment and the oscillator. In the master equation approach the weak coupling leads to a separation of time scales of the environment evolution and the evolution of the density matrix, resulting in a Markovian master equation. Here the same argument is used in the frequency domain: The width of the oscillator spectral function is small on the scale of the frequency dependence of the environment correlation functions.

The current-voltage characteristic of the junction has been obtained. The stationary state current is seen to consist of two contributions. One contribution is the current through an isolated junction, whereas the other contribution, dependent on the state of the oscillator, stems from oscillator assisted tunneling. The expression obtained for the dc-current is in accordance with the result previously derived using a master equation technique.[17,31] The additional current arising from the influence of the oscillator on the junction transmission, Eq. (4.15), vanishes if the oscillator is in the ground state and the voltage across the junction is smaller than the oscillator frequency. We observe therefore that the zero point fluctuations of the oscillator position do not affect the electric current.

The main part of the paper presents the calculation of the electric noise due to oscillator-assisted tunnelling. The unsymmetrized current-current correlator has been evaluated for arbitrary frequencies, bias voltages, and environment temperatures. The noise spectrum consists of a smooth noise floor and a peaked resonant structure.

As a function of frequency ω, the noise floor varies on the scale $\max(T, V, \Omega)$. The contribution of the oscillator is shown to vanish at positive frequencies when the oscillator is in the ground state and the bias voltage is smaller than the oscillator frequency. The expressions Eqs. (4.27, 4.32) for the second order noise generalize the result obtained by a Markovian master equation derived in Ref. 17. In the Markovian master equation calculation a constant noise floor was obtained, whereas the Green's function calculation presented here captures the frequency dependent noise floor. As expected, in the low frequency limit, we recover the result obtained by the Markovian master equation approach.

The resonant structure in the noise spectrum consists of peaks at zero frequency, $\omega = 0$, the oscillator frequency, $\omega = \pm\Omega$, and twice the oscillator frequency, $\omega = \pm 2\Omega$. The resonances are of nonequilibrium origin, being present only for a finite voltage across the junction, and their intensities at positive and negative frequencies are not related by the detailed balance relation.[25] The peaks at the oscillator frequency have the same height for positive and negative frequencies, whereas the peaks at double the oscillator frequency are asymmetric. The peaks at $\omega = \Omega$ stem from processes involving a single vibrational quantum, and their height is therefore linear in the oscillator occupation number N^*, whereas the processes leading to the peaks at $\omega = 0$ and $\omega = \pm 2\Omega$ involve two quanta and the peak heights are quadratic in N^*. The peaks at positive frequencies vanish in the limit where the bias voltage is smaller than the oscillator frequency and the oscillator is in the ground state, but a dip in the noise spectrum remains for negative frequencies.

For bias voltages smaller than the oscillator frequency, we find no oscillator dependent noise at positive frequencies: Both the oscillator dependent contribution to the noise floor as well as the peaks at positive frequencies disappear from the unsymmetrized noise spectrum, $S^<(\omega)$. The oscillator contribution to the current-current correlator at negative frequencies remains finite even for voltages smaller than the oscillator frequency.

To understand this result we consider a setup for measuring current fluctuations. Lesovik and Loosen[24] as well as Gavish, Levinson and Imry,[25] introduced a damped harmonic oscillator with resonance frequency Ω_0 as a meter of current fluctuations. For linear coupling of the oscillator to the current in the junction, they obtained the deviation of the meter position fluctuations, $\langle X^2\rangle$, from the equilibrium value in terms of the current-current correlators of the junction, $S^<$ and $S^>$ (see Eq. (4.6)), and the meter occupation number, N_{Ω_0},

$$\delta\left\langle \hat{X}^2\right\rangle = A\left[(N_{\Omega_0}+1)\, S^<(\Omega_0) + N_{\Omega_0} S^>(\Omega_0)\right], \quad (6.1)$$

where A is the coupling strength and N_{Ω_0} is the Bose function with temperature T. Note that the argument of the noise spectrum $S^{<,>}$, the oscillator frequency Ω_0 is positive. Let us consider the meter fluctuations in different temperature ranges. For large detector temperatures $T \gg \Omega_0$, and consequently $N_{\Omega_0} \gg 1$, the meter fluctuations are proportional to the symmetrized current-current correlator of the junction, $S^> + S^<$. A *passive* detector – a detector at low temperature $T \ll \Omega_0$, when $N_{\Omega_0} \ll 1$ – measures the unsymmetrized current-current correlator $S^<$ at *positive* frequency. In accordance with these arguments, only the part of $S^<$ at positive frequencies describes physical noise, *i.e.*, noise emitted by a system to the environment.

We can now make sense of our results for the current-current correlator $S^<(\omega)$. A passive detector detects only the positive frequency part of the current-current correlator. Our calculations show zero $S^<$ at positive frequencies in the case when the oscillator is in the ground state, and thus the ground state does not contribute to the noise in the tunnel junction in compliance with general expectations.

The vanishing of the oscillator dependent noise at bias voltages smaller than the oscillator frequency also confirms that the master equation approach to finite frequency noise is only applicable at large voltages, $V \gg \omega$ (see Ref. 17). Non-vanishing noise in the regime of bias voltages smaller than the oscillator frequency, as obtained from the extension of a master equation approach beyond its region of applicability, has to be considered as an artefact of the approximations involved in the derivation of a master equation.

In experiments with nanomechanical resonators utilizing electrical devices as detectors, noise properties can function as a diagnostic tool in determining the state of the resonator. In this paper we presented a calculation of the noise induced by an oscillator in a tunnel junction, valid at arbitrary parameters and thus also in the important region where the oscillator approaches the ground state.

Appendix A: JUNCTION GREEN'S FUNCTIONS

In this section we are going to calculate the junction Green's functions that appeared in the calculation of the properties of the oscillator stationary state, the average current and the noise. The junction Green's functions encountered are

$$\begin{aligned}\Pi^{(+)}_{uu'}(\tau_1,\tau_2) &= -i\left\langle \mathrm{T}_c\left(h_u(\tau_1)h_{u'}(\tau_2)\right)\right\rangle ,\\ \Pi^{(-)}_{uu'}(\tau_1,\tau_2) &= -\left\langle \mathrm{T}_c\left(h_u(\tau_1)j_{u'}(\tau_2)\right)\right\rangle ,\end{aligned} \tag{A1}$$

where the notation was introduced in section II, and the contour times discussed in section III. In terms of the tunneling operators $\mathcal{T}$ and $\mathcal{T}^\dagger$, Eq. (2.3), taken in the interaction picture, we have

$$\begin{aligned}\Pi^{(\pm)}_{uu'}(\tau_1,\tau_2) &= -i\left[\left\langle \mathrm{T}_c\left(\mathcal{T}_u(\tau_1)\mathcal{T}^\dagger_{u'}(\tau_2)\right)\right\rangle\right.\\ &\pm \left.\left\langle \mathrm{T}_c\left(\mathcal{T}^\dagger_u(\tau_1)\mathcal{T}_{u'}(\tau_2)\right)\right\rangle\right] .\end{aligned} \tag{A2}$$

Inserting the tunneling operator from Eq. (2.7) we get the symmetric and antisymmetric combinations of left and right electrode electron Green's functions

$$\begin{aligned}\Pi^{(\pm)}_{uu'}(\tau_1,\tau_2) &= -i\sum_{\mathbf{lr}} u_{\mathbf{lr}}u'_{\mathbf{rl}}\left[G_{\mathbf{l}}(\tau_2-\tau_1)G_{\mathbf{r}}(\tau_1-\tau_2)\right.\\ &\quad\left.\pm G_{\mathbf{r}}(\tau_2-\tau_1)G_{\mathbf{l}}(\tau_1-\tau_2)\right] ,\end{aligned} \tag{A3}$$

where we introduced the electron Green's function for the right and left electrodes, for example

$$G_{\mathbf{l}}(\tau_1,\tau_2) = -i\left\langle \mathrm{T}_c\left(c_{\mathbf{l}}(\tau_1)c^\dagger_{\mathbf{l}}(\tau_2)\right)\right\rangle . \tag{A4}$$

The real time Green's functions of interest can be obtained by standard techniques. For example, suppressing the dependence on the tunneling amplitudes u and u' on the left ,

$$\Pi^<_{(\pm)}(t_1,t_2) = -i\left[\left\langle \mathcal{T}^\dagger_{u'}(t_2)\mathcal{T}_u(t_1)\right\rangle \pm \left\langle \mathcal{T}_{u'}(t_2)\mathcal{T}^\dagger_u(t_1)\right\rangle\right] ,$$

is given in terms of electron Green's functions as

$$\begin{aligned}\Pi^<_{(\pm)}(t_1-t_2) &= -i\sum_{\mathbf{lr}} u_{\mathbf{lr}}u'_{\mathbf{rl}}\left[G^>_{\mathbf{l}}(t_2-t_1)G^<_{\mathbf{r}}(t_1-t_2)\right.\\ &\quad\left.\pm G^>_{\mathbf{r}}(t_2-t_1)G^<_{\mathbf{l}}(t_1-t_2)\right] ,\end{aligned}$$

or in terms of its Fourier transform

$$\begin{aligned}\Pi^<_{(\pm)}(\omega) &= -i\sum_{\mathbf{lr}} u_{\mathbf{lr}}u'_{\mathbf{rl}}\int d\omega_1\left[G^>_{\mathbf{l}}(\omega_1-\omega)G^<_{\mathbf{r}}(\omega_1)\right.\\ &\quad\left.\pm G^>_{\mathbf{r}}(\omega_1-\omega)G^<_{\mathbf{l}}(\omega_1)\right] ,\end{aligned}$$

We can insert the single electron Green's functions

$$\begin{aligned}G^>_{\mathbf{l}}(\omega) &= -2\pi i\left[1-f(\epsilon_{\mathbf{l}})\right]\delta(\omega-\epsilon_{\mathbf{l}}+V),\\ G^<_{\mathbf{l}}(\omega) &= 2\pi i f(\epsilon_{\mathbf{l}})\delta(\omega-\epsilon_{\mathbf{l}}+V)\\ G^>_{\mathbf{r}}(\omega) &= -2\pi i\left[1-f(\epsilon_{\mathbf{r}})\right]\delta(\omega-\epsilon_{\mathbf{r}}),\\ G^<_{\mathbf{r}}(\omega) &= 2\pi i f(\epsilon_{\mathbf{r}})\delta(\omega-\epsilon_{\mathbf{r}})\end{aligned}$$

and obtain

$$\begin{aligned}\Pi^<_{(\pm)}(\omega) &= -2\pi i\sum_{\mathbf{lr}} u_{\mathbf{lr}}u'_{\mathbf{rl}}\left[f(\epsilon_{\mathbf{l}})-f(\epsilon_{\mathbf{r}})\right]\\ &\quad\times\left\{\left[1+n(\epsilon_{\mathbf{r}}-\epsilon_{\mathbf{l}})\right]\delta(\epsilon_{\mathbf{r}}-\epsilon_{\mathbf{l}}+V-\omega)\right.\\ &\quad\left.\pm\, n(\epsilon_{\mathbf{r}}-\epsilon_{\mathbf{l}})\delta(\epsilon_{\mathbf{r}}-\epsilon_{\mathbf{l}}+V+\omega)\right\} ,\end{aligned} \tag{A5}$$

where $n(\omega)$ is the Bose function. For voltages $V \ll E_F$ we obtain the approximate relation

$$\begin{aligned}&\textstyle\sum_{\mathbf{lr}} u_{\mathbf{lr}}u'_{\mathbf{rl}}\left[f(\epsilon_{\mathbf{l}})-f(\epsilon_{\mathbf{r}})\right]\delta(\epsilon_{\mathbf{r}}-\epsilon_{\mathbf{l}}+V)\\ &= V\textstyle\sum_{\mathbf{lr}} u_{\mathbf{lr}}u'_{\mathbf{rl}}\left(-\frac{\partial f(\epsilon_{\mathbf{l}})}{\partial\epsilon_{\mathbf{l}}}\right)\delta(\epsilon_{\mathbf{r}}-\epsilon_{\mathbf{l}}),\end{aligned} \tag{A6}$$

as tunneling amplitudes depend only weakly on their arguments, so that

$$\Pi^<_{(\pm)}(\omega) = iG_{uu'}\left[(V-\omega)\left[1+n(V-\omega)\right]\right. \tag{A7}$$

$$\left.\pm(V+\omega)\,n(V+\omega)\right] \tag{A8}$$

where we introduced the conductance

$$G_{uu'} = 2\pi\sum_{\mathbf{lr}} u_{\mathbf{lr}}u'_{\mathbf{rl}}\left(-\frac{\partial f(\epsilon_{\mathbf{l}})}{\partial\epsilon_{\mathbf{l}}}\right)\delta(\epsilon_{\mathbf{r}}-\epsilon_{\mathbf{l}}). \tag{A9}$$

The retarded, advanced and Keldysh bubbles

$$\begin{aligned}\Pi^R_{(\pm)}(t_1-t_2) &= -i\theta(t_1-t_2)\left[\left\langle\left[\mathcal{T}_u(t_1),\mathcal{T}^\dagger_{u'}(t_2)\right]\right\rangle\right.\\ &\quad\left.\pm\left\langle\left[\mathcal{T}^\dagger_u(t_1),\mathcal{T}_{u'}(t_2)\right]\right\rangle\right]\\ \Pi^A_{(\pm)}(t_1-t_2) &= -i\theta(t_2-t_1)\left[\left\langle\left[\mathcal{T}_u(t_1),\mathcal{T}^\dagger_{u'}(t_2)\right]\right\rangle\right.\\ &\quad\left.\pm\left\langle\left[\mathcal{T}^\dagger_u(t_1),\mathcal{T}_{u'}(t_2)\right]\right\rangle\right]\\ \Pi^K_{(\pm)}(t_1-t_2) &= -i\left[\left\langle\left\{\mathcal{T}_u(t_1),\mathcal{T}^\dagger_{u'}(t_2)\right\}\right\rangle\right.\\ &\quad\left.\pm\left\langle\left\{\mathcal{T}^\dagger_u(t_1),\mathcal{T}_{u'}(t_2)\right\}\right\rangle\right]\end{aligned} \tag{A10}$$

can be calculated in a similar way to give

$$\Pi^{(-)}_{uu'}(\omega) = -i\begin{pmatrix}\Pi^R_{(-)}(\omega) & \Pi^K_{(-)}(\omega)\\ 0 & \Pi^A_{(-)}(\omega)\end{pmatrix} \tag{A11}$$

with

$$\begin{aligned}\Pi^R_{(-)}(\omega) &= -iG_{uu'}V, + R^{(-)}_{uu'}(\omega)\\ \Pi^K_{(-)}(\omega) &= -2iG_{uu'}\Delta_V(\omega)\end{aligned} \tag{A12}$$

and the advanced part given by $\Pi^A_{(-)}(\omega) = \left[\Pi^A_{(-)}(\omega)\right]^*$, with the functions $S_V(\omega)$ and $\Delta_V(\omega)$ defined in Eqs. (3.6, 4.14) respectively, and similarly $\Pi^{(+)}_{uu'}(\omega)$ is given by Eq. (3.5).

The real parts of the retarded $\Pi^{(\pm)}_{uu'}(\omega)$ are given by

$$R^{(\pm)}_{uu'}(\omega) = 2\pi\sum_{\mathbf{lr}} u_{\mathbf{lr}}u'_{\mathbf{rl}}\left[f(\epsilon_{\mathbf{l}})-f(\epsilon_{\mathbf{r}})\right](P_-\pm P_+) \tag{A13}$$

with

$$P_{\pm} = \frac{1}{\epsilon_{\mathbf{l}} - \epsilon_{\mathbf{r}} \pm V + \omega}. \quad \text{(A14)}$$

To estimate the real parts, we ignore the weak energy dependence of the transmission amplitudes are constants up to a cut-off frequency of the order of the Fermi energy. For frequencies and voltages much smaller than the cutoff frequency, $\omega, V \ll E_F$, we can approximate the reactive parts of the response functions Eq. (A13) to give

$$\begin{aligned} R_{(-)}(\omega) &\sim G_{uu'} V \frac{\omega}{E_F} \\ R_{(+)}(\omega) &\sim G_{uu'} E_F. \end{aligned} \quad \text{(A15)}$$

This concludes our calculation of the junction Green's functions.

Appendix B: FOURTH ORDER CURRENT-CURRENT CORRELATOR

In this section we are going to evaluate the fourth order current-current correlator, Eq. (4.21),

$$\begin{aligned} & S_{(4)}(\tau_3,\tau_4) = -I^2 \\ & -\tfrac{1}{2}\int_c d\tau_1 d\tau_2 \left\langle \mathrm{T}_c \left(\hat{H}_T(\tau_1)\hat{H}_T(\tau_2)\hat{I}(\tau_3)\hat{I}(\tau_4) \right) \right\rangle, \end{aligned} \quad \text{(B1)}$$

where the average current, I, already was evaluated to second order in Eq. (4.8). We then insert the tunneling Hamiltonian Eq. (2.6) and the current operator Eq. (2.8). The electronic degrees of freedom separate from the oscillator degrees of freedom and the fourth order current-current correlator splits into terms proportional to v^4, terms proportional to v^2w^2 and terms proportional w^4, so that

$$S_{(4)}(3,4) = S_{v^4}(3,4) + S_{v^2w^2}(3,4) + S_{w^4}(3,4), \quad \text{(B2)}$$

where the short notation $S_{(4)}(\tau_3,\tau_4) = S_{(4)}(3,4)$, etc. is introduced. The contribution corresponding to the isolated junction is

$$S_{v^4}(3,4) = -\frac{1}{2}\int_c d\tau_1 d\tau_2\, C_{vvvv}(1,2,3,4) - I_{vv}^2, \quad \text{(B3)}$$

where the current I_{vv} is given by Eq. (4.11). The contribution linear in the oscillator Green's function is

$$\begin{aligned} S_{v^2w^2}(3,4) &= -\frac{i}{2}\int_c d\tau_1 d\tau_2 C_{wvwv}(1,2,3,4) \\ &\times [D_0(1,3) + D_0(1,4) + D_0(2,3) + D_0(2,4)] \\ &- \frac{i}{2}\int_c d\tau_1 d\tau_2\, C_{vvww}(1,2,3,4) \\ &\times [D_0(3,4) + D_0(1,2)] - 2I_{vv}I_{ww}, \end{aligned} \quad \text{(B4)}$$

with the current I_{ww} given by Eq. (4.10). The contribution quadratic in the oscillator Green's functions

$$\begin{aligned} S_{w^4}(3,4) &= \frac{1}{2}\int_c d\tau_1 d\tau_2\, C_{wwww}(1,2,3,4) \\ &\times [D_0(1,2)D_0(3,4) + D_0(1,3)D_0(2,4) \\ &+ D_0(2,3)D_0(1,4)] - I_{ww}^2, \end{aligned} \quad \text{(B5)}$$

where Wick's theorem was applied to the oscillator operators, introducing the free oscillator Green's function

$$D_0(1,2) = -i \left\langle \mathrm{T}_c \left(\hat{x}(\tau_1)\hat{x}(\tau_2) \right) \right\rangle. \quad \text{(B6)}$$

In order to correctly include the influence of the environment on the oscillator, the free oscillator Green's should be replaced by the full Green's function, D. The full Green's function includes damping and was calculated in section III. In the following this replacement will always be understood.

The fourth order junction correlation function is defined as

$$C_{u_1u_2u_3u_4}(1,2,3,4) = \left\langle \mathrm{T}_c \left(h(1)h(2)j(3)j(4) \right) \right\rangle, \quad \text{(B7)}$$

where $u_i = v, w$ and $h(1)$ is short for $h_{u_1}(\tau_1)$, Eq. (2.7), and similarly $j(1)$ for $j_{u_1}(\tau_1)$, Eq. (2.9). Now we evaluate the junction correlation function Eq. (B7). It can be expressed in terms of the tunneling operators, Eq. (2.7), as

$$\begin{aligned} C_{u_1u_2u_3u_4}(1,2,3,4) &= \left[\left\langle \mathrm{T}_c \left(\mathcal{T}^\dagger(1)\mathcal{T}(2)\mathcal{T}(3)\mathcal{T}^\dagger(4) \right) \right\rangle \right. \\ &+ \left. (3 \leftrightarrow 4) - (2 \leftrightarrow 4) \right] + h.c. \end{aligned} \quad \text{(B8)}$$

where we used the short notation $\mathcal{T}(1)$ for $\mathcal{T}_{u_1}(\tau_1)$ and similarly for the hermitian conjugate. It is sufficient to evaluate the correlation function of the tunneling operators

$$\begin{aligned} & \left\langle \mathrm{T}_c \left(\mathcal{T}^\dagger(1)\mathcal{T}(2)\mathcal{T}(3)\mathcal{T}^\dagger(4) \right) \right\rangle = \\ & \sum_{\mathbf{l}_1\mathbf{r}_1,\ldots\mathbf{l}_4\mathbf{r}_4} u_1u_2u_3u_4 \left[G_{\mathbf{l}}(1,2)G_{\mathbf{l}}(4,3) - G_{\mathbf{l}}(1,3)G_{\mathbf{l}}(4,2) \right] \\ & \qquad \times \left[G_{\mathbf{r}}(2,1)G_{\mathbf{r}}(3,4) - G_{\mathbf{r}}(3,1)G_{\mathbf{r}}(2,4) \right] \end{aligned} \quad \text{(B9)}$$

where we suppressed the labels $\mathbf{l}$ and $\mathbf{r}$ on the transmission amplitudes and Wick's theorem was applied to the electron correlation functions, using the short notation

$$\begin{aligned} G_{\mathbf{l}}(1,2) &= -i \left\langle \mathrm{T}_c \left(c_{\mathbf{l}_1}(\tau_1)c^\dagger_{\mathbf{l}_2}(\tau_2) \right) \right\rangle \\ G_{\mathbf{r}}(1,2) &= -i \left\langle \mathrm{T}_c \left(d_{\mathbf{r}_1}(\tau_1)d^\dagger_{\mathbf{r}_2}(\tau_2) \right) \right\rangle. \end{aligned} \quad \text{(B10)}$$

The junction correlation function can be split into two terms,

$$\begin{aligned} C_{u_1u_2u_3u_4}(1,2,3,4) &= \tilde{C}_{u_1u_2u_3u_4}(1,2,3,4) \\ &+ \bar{C}_{u_1u_2u_3u_4}(1,2,3,4), \end{aligned} \quad \text{(B11)}$$

where one part, $\tilde{C}$, contains all terms where pairs of left and right side electron Green's functions have the same time arguments

$$\begin{aligned} \tilde{C}_{u_1u_2u_3u_4}(1,2,3,4) &= -\Pi^{(+)}_{u_1u_2}(1,2)\Pi^{(+)}_{u_3u_4}(3,4) \\ &+ \Pi^{(-)}_{u_1u_3}(1,3)\Pi^{(-)}_{u_2u_4}(2,4) \\ &+ \Pi^{(-)}_{u_1u_4}(1,4)\Pi^{(-)}_{u_2u_3}(2,3), \end{aligned} \quad \text{(B12)}$$

where we used the short notation $\Pi^{(\pm)}_{u_1u_2}(\tau_1,\tau_2) = \Pi^{(\pm)}_{u_1u_2}(1,2)$ for the junction correlation functions, Eq. (A3), corresponding to bubble diagrams depicted in Fig. 3 and also that $\langle \mathrm{T}_c \left(h_u(\tau_1) h_{u'}(\tau_2) \right) \rangle = -\langle \mathrm{T}_c \left(j_u(\tau_1) j_{u'}(\tau_2) \right) \rangle$. This term can also be obtained by naively applying Wick's theorem to the junction operators in Eq. (B7). The remaining part of the fourth order junction correlation function is

$$\bar{C}_{u_1 u_2 u_3 u_4}(1,2,3,4) = 2 \sum_{\mathbf{l}_1 \mathbf{r}_1, \ldots, \mathbf{l}_4 \mathbf{r}_4} u_1 u_2 u_3 u_4 \left[G_{\mathbf{l}}(1,3) G_{\mathbf{l}}(4,2) G_{\mathbf{r}}(4,1) G_{\mathbf{r}}(3,2) + h.c. \right], \tag{B13}$$

where we used that the junction correlation function always appears under integration over τ_1 and τ_2, either alone, or together with combinations of oscillator Green's functions that are symmetric under the exchange $\tau_1 \leftrightarrow \tau_2$ and therefore only that part of the integrand $\bar{C}$ remains which is also symmetric under the exchange $\tau_1 \leftrightarrow \tau_2$. The non-bubble contribution to the junction correlation function can be seen in diagrammatic form as the solid lines in Fig. 4.

Since we have split the fourth order junction correlation function into two parts, Eq. (B11), we also divide the current-current correlators Eqs. (B3, B4, B5) into one part, $\tilde{S}$, containing the bubble contribution $\tilde{C}$ and another part, $\bar{S}$, containing the contribution $\bar{C}$

$$S_{u_1^2 u_2^2}(3,4) = \tilde{S}_{u_1^2 u_2^2}(3,4) + \bar{S}_{u_1^2 u_2^2}(3,4), \quad u_i = v, w. \tag{B14}$$

Let us consider first the contribution containing bubbles. Note that the contribution from the average current squared is always of the bubble kind. The bubble contribution not containing any oscillator Green's function vanishes, since the terms from the fourth order current-current correlator cancel with the terms from the current squared. The contribution linear in the oscillator Green's function is

$$\begin{aligned} \tilde{S}_{v^2 w^2}(3,4) &= -i \int_c d\tau_1 d\tau_2 \Pi^{(-)}_{vw}(2,3) \Pi^{(-)}_{vw}(1,4) \\ &\times \left[D(1,3) + D(1,4) + D(1,2) + D(3,4) \right], \end{aligned} \tag{B15}$$

where we used that the integrand is symmetric under exchange of $\tau_1 \leftrightarrow \tau_2$. We return to real time integration and obtain after Fourier transformation the expression in Eq. (B20).

The contribution quadratic in the oscillator Green's function is after inserting Eq. (B12) in Eq. (B5)

$$\begin{aligned} \tilde{S}_{w^4}(3,4) &= -\int_c d\tau_1 d\tau_2 \Pi^{(+)}_{ww}(1,2) \Pi^{(+)}_{ww}(3,4) D(1,3) D(2,4) \\ &+ \int_c d\tau_1 d\tau_2 \Pi^{(-)}_{ww}(2,3) \Pi^{(-)}_{ww}(1,4) \\ &\times \left[D(1,2) D(3,4) + D(1,3) D(2,4) \right], \end{aligned} \tag{B16}$$

where products of the first term in Eq. (B5) with the first term in Eq. (B12), corresponding to disconnected diagrams, vanish under integration over τ_1 and τ_2. Also products of the second term in Eq. (B5) with the second term in Eq. (B12) together with the third term in Eq. (B5) multiplied by the third term in Eq. (B12) cancel the current squared. The contribution containing $\Pi^{(+)}$ is already included in the second order noise $S^<_{ww}(\omega)$, where we replaced the free oscillator Green's function with the full oscillator Green's function. Calculating the component $S^<$ we obtain upon Fourier transformation the expression in Eq. (B22).

What remains is to evaluate the non-bubble contribution $\bar{S}$. We start with the terms that do not contain any oscillator Green's functions. Inserting the junction correlation function, Eq. (B13), in the expression for the current-current correlator, Eq. (B3), we get

$$\bar{S}_{v^4}(3,4) = -\int_c d\tau_1 d\tau_2 \sum_{\mathbf{l}_1 \mathbf{r}_1} \sum_{\mathbf{l}_2 \mathbf{r}_2} v^4 \left[G_{\mathbf{l}}(3,1) G_{\mathbf{l}}(4,2) G_{\mathbf{r}}(1,4) G_{\mathbf{r}}(2,3) + h.c. \right], \tag{B17}$$

where we suppressed the indices on the transmission amplitudes. Similarly, inserting the junction correlation function into Eq. (B4) we get for the terms linear in the oscillator Green's function

$$\begin{aligned} \bar{S}_{v^2 w^2}(3,4) &= -i \int_c d\tau_1 d\tau_2 \sum_{\mathbf{l}_1 \mathbf{r}_1} \sum_{\mathbf{l}_2 \mathbf{r}_2} v^2 w^2 \left[G_{\mathbf{l}}(3,1) G_{\mathbf{l}}(4,2) G_{\mathbf{r}}(1,4) G_{\mathbf{r}}(2,3) + h.c. \right] \\ &\times \left[D(3,4) + 2D(1,3) + 2D(2,4) + D(1,2) \right]. \end{aligned} \tag{B18}$$

Finally the terms quadratic in the oscillator Green's function are obtained by inserting the junction correlation function into Eq. (B5)

$$\bar{S}_{w^4}(3,4) = \int_c d\tau_1 d\tau_2 \sum_{\mathbf{l}_1\mathbf{r}_1}\sum_{\mathbf{l}_2\mathbf{r}_2} w^4 \left[G_{\mathbf{l}}(3,1)G_{\mathbf{l}}(4,2)G_{\mathbf{r}}(1,4)G_{\mathbf{r}}(2,3) + h.c.\right] \times \left[D(1,2)D(3,4) + 2D(1,3)D(2,4)\right]. \tag{B19}$$

We can consider a typical term in the non-bubble contribution to the current-current correlator, e.g. the term in Eq. (B18) that combines the electronic Green's functions with the second oscillator Green's function. It is given in Eq. (B23) and also depicted in Fig. 4.

We can continue evaluating the contributions to the noise spectrum. Let us consider, for example, the contribution containing bubbles that is linear in the oscillator Green's function. Starting from Eq. (B15), introducing labels for the integration contours and splitting the contour in forward and backward contour we obtain after Fourier transformation

$$\begin{aligned}\tilde{S}^{<}_{v^2w^2}(\omega) = -i\sum_{kl} s^{kl} \Big(&\Pi^{l1}_{(-)}(\omega)\Pi^{k2}_{(-)}(\omega)D^{kl}(\omega)\\ &+\Pi^{l1}_{(-)}(0)\Pi^{k2}_{(-)}(0)D^{12}(\omega)\\ &+\Pi^{l1}_{(-)}(\omega)\Pi^{k2}_{(-)}(0)D^{k2}(\omega)\\ &+\Pi^{l1}_{(-)}(0)\Pi^{k2}_{(-)}(\omega)D^{k1}(\omega)\Big),\end{aligned} \tag{B20}$$

where, as discussed in section IV A, the full oscillator Green's function appears, accounting for the influence of the interaction with the tunnel junction and the matrix

$$s^{kl} = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \tag{B21}$$

introduces the correct signs for forward and backward contour. The contribution to the noise, Eq. (B20), can be represented by the four diagrams shown in Fig. 1. Performing the summation over the contour indices k and l, and inserting the bubbles and Green's functions we obtain Eq. (4.34).

Similarly, the contribution containing two oscillator Green's functions according to Eq. (B16) becomes

$$\begin{aligned}\tilde{S}^{<}_{w^4}(\omega) = \sum_{kl} s^{kl} \int d\omega_1 \Big(&\Pi^{l1}_{(-)}(\omega_1)\Pi^{k2}_{(-)}(\omega_1)D^{kl}(\omega_1)D^{21}(\omega-\omega_1)\\ &+\Pi^{l1}_{(-)}(\omega-\omega_1)\Pi^{k2}_{(-)}(\omega_1)D^{2l}(\omega-\omega_1)D^{k1}(\omega_1).\end{aligned} \tag{B22}$$

The fourth order contribution, $\tilde{S}^{<}_{w^4}(\omega)$, is displayed in diagrammatic form in Fig. 2. Doing the summation and inserting the bubbles and oscillator Green's functions we obtain Eq. (4.35), where we neglected a non-resonant contribution that is small compared to the second order contributions to the noise floor, Eqs. (4.32, 4.27).

Let us turn our attention to the parts of the noise spectrum, $\bar{S}$, not containing bubbles. All non-bubble contributions, Eqs. (B17, B18, B19), are of the non-resonant kind, i.e. oscillator Green's functions always appear under frequency integration. A typical non-bubble contribution to the noise spectrum, e.g. one of the terms in Eq. (B18), is shown in Fig. 4 and given by

$$\bar{s}^{<}_{v^2w^2}(\omega) = \int d\omega_1 d\omega_2 \sum_{kl} s^{kl} \sum_{\mathbf{l}_1\mathbf{r}_1}\sum_{\mathbf{l}_2\mathbf{r}_2} v^2w^2 G^{1k}_{\mathbf{l}_1}(\omega_1-\omega_2)G^{2l}_{\mathbf{l}_2}(\omega_1-\omega)G^{k2}_{\mathbf{r}_1}(\omega_1)G^{l1}_{\mathbf{r}_2}(\omega_1-\omega)D^{k1}(\omega_2). \tag{B23}$$

Since there are no resonant peaks present in Eqs. (B17, B18, B19), they have to be compared to the second order contribution to the noise floor, Eqs. (4.27, 4.32) and since they are always of higher order in tunneling they can be neglected.

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[31] The additional pumping contribution to the current found in Ref. 17 is not present here, since for simplicity we take the junction to be symmetric implying real transmission matrix elements.
[32] We have not explicitly included terms in the self energy, that correspond to a shift in the equilibrium position of the oscillator. The shift can be absorbed by measuring the oscillator coordinate from the new equilibrium position. We have to keep in mind, though, that the coordinate independent part of the tunneling amplitude, v_{rl}, has to be changed accordingly.