A bit confusing. I welcome feedback on this solution. / Peter

2.3.4 The Allee effect

a) *Show that the effective growth rate is highest for intermediate N in the following model:*

$$\dot{N}/N = r - a(N-b)^2.$$

This is an inverted parabola with maximum at $N = b$.

b) We have

$$f(N) = Nr - aN(N-b)^2 = Nr - ab^2N + 2abN^2 - aN^3,$$

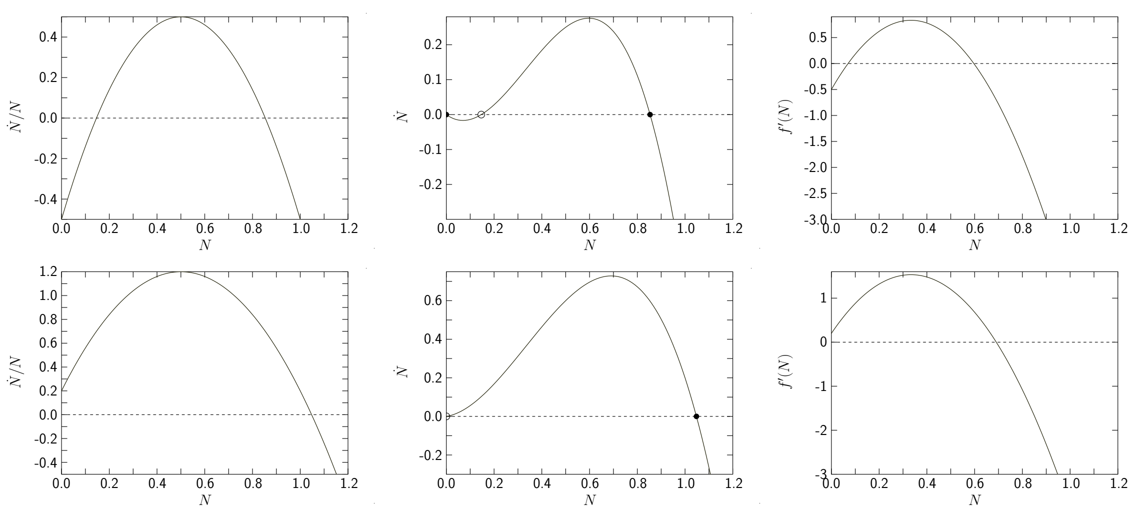
which gives

$$f'(N) = r - ab^2 + 4abN - 3aN^2.$$

Fixed points:

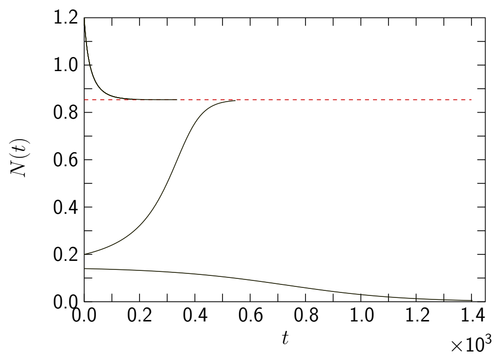
1. $N = 0$, gives $f' = r - ab^2$, which is stable if $ab^2 > r$.
2. $r = a(N-b)^2$ which gives $N^*_\pm = b \pm \sqrt{r/a}$.

The figures below are from taking $a = 4$, $b = 1/2$ and for two different r: Figures above: $r = 0.5$ and below: $r = 1.2$.



The figures to the left show the two different possible behaviors: above: $N^*_- > 0$ and below $N^*_- < 0$ (which is not a valid fixed point). In the first case the trivial fixed point is stable (leading to extinction), in the second case it is unstable.

c) Consider the first case, for $r = 0.5$. This leads to extinction for $N < N_-^* \approx 0.15$. Otherwise the long time solution is N_+^*.



d) A difference of the $N(t)$ shown above compared to the solutions from the logistic equation is that we can here get extinction if N starts from a low value.