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New and old problems

1 Indoor air humidity

Since the cold outdoor air contains a very small amount of water the indoor air becomes very dry during winter. Assume an outdoor temperature of 0° C and 100% relative humidity and indoor temperature of 20° C.

- a) Calculate the relative humidity of the indoor air. Assume that the outdoor air is quickly circulating to the inside, as is common of modern buildings.
- b) Sara wants to rise the humidity in her appartment of 23 m^2 and 2.4 m from floor to ceiling by 50 percentage (which could e.g. be from 5% to 55%) by boiling water. How much water needs to be boiled away? How much energy is needed in this process?

2 Opening refrigerator door

You open a kitchen refrigerator and close it again. In this process the volume fraction $\epsilon = 0.1$ of the cold air in the refrigerator is replaed by warm air of the room. After a short pause you try to open the refrigerator again and find that you then need to apply a large force. Calculate this force!

Take the room temperature to be $T_1 = 20^{\circ}$ C, the temperature in the refrigerator to be $T_2 = 0^{\circ}$ C and the refrigerator door to have an area of A = 1m².

3 Chemical potential

The heat capacity of a degenerate electron gas at constant volume may be written as

$$C_v = aN\left(\frac{V}{N}\right)^{2/3}T$$

where a is a factor involving only universal constants of physics. Determine how the chemical potential of the gas depends on temperature.

4 Heat engine

Consider a heat engine with a cycle consisting of three steps:

1. adabatic compression from V_1 to V_2 ,

- 2. isothermal expansion back to volume V_1 ,
- 3. pressure relaxation to the initial state at constant volume.

Find the efficiency of such an engine. Assume that the working substance is a two-atomic gas with "frozen out" oscillation degrees of freedom.

5 Phase transition in ³He

Consider the phase transition between solid and liquid ³He at low temperatures. At absolute zero the transition happens at a pressure P_0 , the solid phase corresponds to $P > P_0$, the liquid phase is at $P < P_0$. The entropy per mole of the liquid phase is given by $S_l = \alpha T$ whereas the entropy of the solid phase may be taken to be a constant $S_s = S_0$. The volume per mole of both the liquid and the solid phases may be taken to be a constant with $\Delta V = V_l - V_s = V_0 > 0$.

Determine the phase boundary curve and the critical pressure, P_c , below which solid ³He cannot exist. *Hint:* Consider Fig. 5.13 in the textbook.

6 Moving mass Note that this is an unusually difficult problem.

Consider a thermally isolated chamber of total volume 2V filled with a one-atomic gas with initial temperature T_i . The volume is separated into two parts by a movable plate of surface area A and an unknown mass M as shown in the figure. The plate allows themrla interactino between the upper and the lower parts of the chamber. We have the same amount of particles N in each part. Initially the plate is fixed and the volumes of both parts are the same and equal to V. We then let the plate move under the action of gravity. After a transitional period the system reaches mechanical and thermal equilibrium with a new temperature of the gas and new volumes, $V_1 = V/2$ for the lower part and $V_2 = 3V/2$ for the upper part. Find the mass M of the plate. Neglect the potential energy of the gas. *Hint: Think about energy conservation in the system*.

7 Entropy change

Consider a chamber of total volume 2V separated into two parts of volume $V_1 = V/2$ and $V_2 = 3V/2$. The parts are filled with one-atomic

gases A and B, respectively, which are initially in mechanical, but not thermal equilibrium with each other. There is an equal amount of atoms for each gas, $N_A = N_B = N$. We then remove the plate and thus allow thermal and diffusive interaction between gases. Find the increase in entropy of the system.

8 Chemical potential of a crystal

According to the Debye model, the heat capacity of a crystal at low temperatures may be written as

$$C_v = aNk_B \left(\frac{T}{T_D}\right)^3,$$

where a is a numerical factor and T_D is a constant known as the Debye temperature. Find how the chemical potential of the crystal depends on the temperature.

9 Engine

Consider an engine cycle with three steps,

- 1. adabatic compression from V_1 to V_2 ,
- 2. isobaric expansion back to volume V_1 ,
- 3. pressure relaxation to the initial state at constant volume.

Find the efficiency of such an engine. Assume that the working substance is a two-atomic gas with "frozen out" oscillation degrees of freedom.

10 Modified van der Waal's model

Imagine the modified van der Waal's model

$$\left(P+a\frac{N^3}{V^3}\right)(V-bN)=Nk_BT,$$

where a and b are some constants. Consider an expansion of such a gas from V = 5bN to V = 9bN at constant N and T and determine the change in Gibbs free energy of the gas.

11 Leaking container

A container of volume V is filled with a gas of temperature T, initial total number of particles N_0 , and mass m for each particle. We make an extremely small hole of area A in the container and the gas starts leaking out slowly due to random collisions of gas particles with the wall (or rather, the hole). How does the number of gas particles N decrease with time?

Hint: Find how many particles dN that pass through the hole in a time interval dt and solve the equation for dN/dt. Take the average thermal velocity in one direction to be $\langle v_x \rangle \approx \langle v_x^2 \rangle^{1/2}$.

12 Heat capacity

Imagine a certain material with heat capacity given by $C_v = aT^3$, where a is some constant. Find how the entropy of this material depends on the energy, U.