Umeå Universitet
Department of Physics
Peter Olsson

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New and old problems

1 Indoor air humidity

a) The relative humidity becomes 26.6%.

b) To increase the relative humidity by 50% we need to boil away 0.468 kg which requires 1.06 MJ of energy.

2 Opening refrigerator door

$$P_{\mathrm{atm}} A\epsilon \left(1 - \frac{T_2}{T_1}\right)$$

3 Chemical potential

$$\mu(T) = -\frac{a}{6}\left(\frac{V}{N}\right)^{2/3} T^2.$$

Basics We can determine S and U from

$$C_v = T\left(\frac{\partial S}{\partial T}\right)_V \quad \Rightarrow \quad S = \int \frac{C_V}{T} dT = aN^{1/3}V^{2/3}T,$$

and

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V \quad \Rightarrow \quad U = \int C_V dT = aN^{1/3}V^{2/3}\frac{T^2}{2}.$$

Incorrect approach One can then try

$$\mu = -T\left(\frac{\partial S}{\partial N}\right)_{U,V} = -\frac{a}{3}\left(\frac{V}{N}\right)^{2/3} T^2,$$

or

$$\mu = \left(\frac{\partial U}{\partial N}\right)_{S,V} = \frac{a}{6}\left(\frac{V}{N}\right)^{2/3} T^2,$$

but they give different answers which both are wrong.

Hint The reason that this is incorrect is that we have not kept U constant in the first case and S constant in the second. To keep U constant when we differentiate S the temperature has to change as N changes. Formally differentiating the expression for U gives

$$dU = a\frac{1}{3}N^{-2/3}V^{2/3}\frac{T^2}{2}dN + aN^{1/3}V^{2/3}TdT,$$

and taking $dU = 0$ leads to

$$\frac{1}{3}N^{-2/3}\frac{T^2}{2}dN = -N^{1/3}dT \quad \Rightarrow \quad dN = -6\frac{N}{T}dT,$$

whereas a similar approch with $dS = 0$ leads to $dN = -3\frac{N}{T}dT$. With this approach there will be two terms in $(\partial S/\partial N)$ and $(\partial U/\partial N)$ and both paths will lead to the correct answer.

We then use

$$dU = \left(\frac{\partial U}{\partial N}\right)_{VT} dN + \left(\frac{\partial U}{\partial T}\right)_{VN} dT,$$

which gives

$$\begin{aligned}
\left(\frac{\partial U}{\partial N}\right)_{SV} &= \left(\frac{\partial U}{\partial N}\right)_{VT} + \left(\frac{\partial U}{\partial T}\right)_{VN}\left(\frac{\partial T}{\partial N}\right)_{SV} \\
&= a\frac{1}{3}N^{-2/3}V^{2/3}\frac{T^2}{2} - aN^{1/3}V^{2/3}T\frac{T}{3N} \\
&= \left(\frac{a}{6} - \frac{a}{3}\right)\left(\frac{V}{N}\right)^{2/3} T^2 \\
&= -\frac{a}{6}\left(\frac{V}{N}\right)^{2/3} T^2
\end{aligned}$$

4 Heat engine

$$\eta = 1 - \frac{f}{2}\frac{1-(V_2/V_1)^{\gamma-1}}{\ln(V_1/V_2)}.$$

5 Phase transition in ^{3}He

$$P_c = P_0 - \frac{S_0^2}{2V_0\alpha}.$$

6 Moving mass *Note that this is an unusually difficult problem.*

$$Mg = \frac{12}{7} \frac{N k_B T_i A}{V}$$

7 Entropy change

$$\Delta S = n k_B \ln \left(\frac{128}{27} \sqrt{3} \right)$$

8 Chemical potential of a crystal

$$\mu = -\frac{a k_B T^4}{12 T_D^3}$$

9 Engine

$$\eta = 1 - \frac{f/2}{f/2 - 1} \frac{(V_2/V_1)^{1/\gamma} - 1}{1 - V_2/V_1}.$$

10 Modified van der Waal's model

My result

$$\Delta G = -N k_B \ln 2 - \frac{N k_B T}{8} + \frac{28 a N}{27 \cdot 25 b^2}.$$

Vitaly Bychkov:

$$\Delta G = -N k_B \ln 2 - \frac{N k_B T}{8 N b} + \frac{9 a N}{128 b^2}.$$

11 Leaking container

$$N = N_0 e^{-t/\tau}, \quad \tau = 2 \frac{V}{A} \sqrt{\frac{M}{k_B T}}$$

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12 Heat capacity

$$S = \frac{1}{3} a^{1/4} (4U)^{3/4}.$$