Comments to the Stirling lab

In the course, as well as in the textbook, we use the convention that Q and W are positive when energy is going into the system. That is however not done consistently in the lab instruction and the purpose with this note is to present the calculations of the efficiency of the Stirling engine in a way that I find acceptable.

The only exception to the rule that Q and W are positive when energy is going into the system is the conventions for heat engines and refrigerators where the energy flows. We have then introduced Q_c, Q_h, and W_e ('e' for engine—my own notation) that are specially defined—they are always positve. In the heat engine we have $Q_c = -Q$ and $W_e = -W$, and in the refrigerator we instead have $Q_h = -Q$. Another thing that makes the lab instruction for the Stirling engine confusing is the use of absolute values. There is no need to do that since we always know the sign of the heat flows.

Below are the calculations of the efficiency of the Stirling engine, done in a consistent way and without any absolute values.

This should be instead of the text on the middle of page 2:
The work done on the gas as it changes from state A to B is $W_{A-B} = -\int_A^B p dV$. In an ideal gas, where the internal energy $U = f(T)$, we know according to the first law of thermodynamics that $\Delta U = Q + W = 0$ in an isothermal process. The first law of thermodynamics then becomes $Q_{A-B} = -W_{A-B}$.

According to **figure 1**:

1 → 2 The gas is isothermally compressed at the temperature T_c and the work $W_{1\to2} = nRT_c \ln V_a/V_b > 0$ is done on the gas while heat flows out of the system, $Q_{1\to2} = -W_{1\to2} < 0$. The heat $Q_c = -Q_{1\to2}$ is leaving the system.

2 → 3 The gas is heated at constant volume (isochoric) to the temperature T_h by supplying the heat $Q_{23} = Q_R$. No work is done.

3 → 4 The gas expands isothermally at the temperature T_h and does work on the environment which means that the work done *on the system* is

negative, $W_{34} = -nRT_h \ln(V_a/V_b) < 0$. The heat $Q_h = Q_{34} = -W_{34} > 0$ is absorbed during this process.

$4 \to 1$ The gas is cooled at constant volume to the temperature T_c. No work is done. The heat is $Q_{41} = -Q_R < 0$, which means that heat flows out of the system.

Since $V_a > V_b$

$$Q_c = -Q_{12} = nRT_c \ln V_a/V_b,$$

$$Q_h = Q_{34} = nRT_h \ln V_a/V_b.$$

For processes $2 \to 3$ and $4 \to 1$ we note that V is constant and

$$\left(\frac{\partial U}{\partial T}\right)_V = C_V.$$

We therefore get $dU = C_V dT$ and (since $dU = Q$ as $W = 0$)

$$Q_{41} = C_V(T_c - T_h),$$

and

$$Q_{23} = C_V(T_h - T_c),$$

which gives $Q_{41} + Q_{23} = 0$.

The efficiency can be written

$$\eta = \frac{W_e}{Q_h} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h} = 1 - \frac{T_c}{T_h}.$$