Interesting problems on air and pressure,

1 Energy in the Sauna

Determine how/if the thermal energy of the air in a Sauna changes when the Sauna is warmed up. Also try to explain this in non-technical terms.

Solution: From $U = \frac{f}{2}PV$ we may conclude that the energy doesn't change if PV is a constant, which is certainly the case in a Sauna.

The energy *per molecule* does indeed increase but the number of molecules decreases by the same proportion when the temperature increases. The energy needed to heat up the Sauna goes into the environment outside the sauna and to the walls, floor, and ceiling of the sauna.

2 Density of the air

Use the ideal gas law to calculate the density of air at NTP (normal temperature and pressure) which is defined by NIST (National Institute of Standards and Technology) to be a temperature of 20°C and an absolute pressure of 1 atm (101.325 kPa). Take the Oxygen content to be ≈21% and assume for simplicity that the rest is Nitrogen.

Solution: The density is

$$\rho = \frac{Nm}{V} = m\frac{P}{k_B T},$$

where m is the average mass of the air molecules, $(0.21 \times 32 + 0.79 \times 28) \times 1.661 \times 10^{-27} = 4.79 \times 10^{-26}$. This gives the density $\rho = 1.200$ kg/m^3.

3 Humid air

What has higher density, dry or humid air?

Solution: Since water molecules are lighter than both Nitrogen and Oxygen the average molecular mass of humid air becomes lower which, according the the calculation above, gives a lower density.

4 Hot air balloon

Calculate the diameter of a spherical balloon (to make it simple) that can lift 1000 kg. Use the density of air calculated above as a main starting point. Assume that the ambient air is at 20°C and that the hot air in the balloon can be 50°C warmer.

Solution: Make use of $\rho_{20} = 1.2$ kg/m^3 and the fact that the density is $\propto 1/T$. The lift force is due to the difference between the mass of the ambient air and the mass of the hot air for the volume of the balloon, which is

$$m = V(\rho_{20} - \rho_{\text{hot}}) = V\rho_{20}\left(1 - \frac{T_{20}}{T_{\text{hot}}}\right).$$

which gives

$$V = \frac{m}{\rho_{20}}\left(1 - \frac{T_{20}}{T_{\text{hot}}}\right)^{-1} = \frac{10^3}{1.2}\frac{1}{0.146} = 8232\text{m}^3,$$

and we find

$$r = \left(\frac{3V}{4\pi}\right)^{1/3} = 12.5\text{m},$$

which gives a diameter of 25m.

5 Pressure in water

At what depth under water is the pressure twice as big as it is on the surface?

Solution: We are looking for the height h of a pillar of water that gives the pressure = 1 atm. With the density of water $\rho = 1000\text{kg/m}^3$, the mass for a pillar of an area A given by $m = \rho V = \rho h A$, the weight is mg and the pressure due to that mass becomes $P = mg/A = \rho g h$. We then get

$$h = \frac{P}{\rho g} = \frac{1.013 \times 10^5}{1e3 \times 9.82} = 10.3\text{m}.$$

6 Air-tight building

Buildings and rooms are not air tight. We here nevertheless assume that a room actually is air tight and determine the increase of pressure that would follow as the temperature of the room increased from 17°C to 27°C, e.g. due to the sun shining in through a window. What would be the force on a window with an area of 1m^2? Would you expect it to break?

Solution: Assume that we start with $P_{\text{low}} = 1$ atm at the lower temperature, $T_{\text{low}} = 290$ K. At the higher temperature T_{high} we get

$$P_{\text{high}} = \frac{T_{\text{high}}}{T_{\text{low}}} P_{\text{low}} \approx 1.034 P_{\text{low}}.$$

Assuming that the outdoor pressure remains the same we find the force on the window due to the pressure difference to be

$$F = A\Delta P = 1 \times 0.034 \times 10^5 = 3400\text{N},$$

which corresponds to the weight of about 340 kg. The window would certainly break!