Problem 4.18

Find the efficiency of the Otto cycle!
Solution by V. Bychkov. Notations from Fig. 4.5.

There are four steps:

$1 \to 2$ adiabatic compression,

$2 \to 3$ ignition at constant volume,

$3 \to 4$ power stroke—adiabatic expansion,

$4 \to 1$ exhaust at constant volue.

Solution

The efficiency is as usual

$$e = 1 - \frac{Q_c}{Q_h}.$$

The energy of an ideal gas:

$$U = \frac{f}{2} N k_B T = \frac{f}{2} PV. \tag{1}$$

From the heat flowing into the system

$$Q_h = Q_{2\to 3} = U_3 - U_2 = \frac{f}{2} V_2 (P_3 - P_2).$$

and the heat flowing out

$$Q_c = -Q_{4\to 1} = -(U_1 - U_4),$$

which therefore gives

$$Q_c = U_4 - U_1 = \frac{f}{2} V_1 (P_4 - P_1),$$

we get

$$\frac{Q_c}{Q_h} = \frac{V_1}{V_2} \frac{P_4 - P_1}{P_3 - P_2}. \tag{2}$$

To relate the pressure differences to the volumes we then make use of the relations for the adiabatic steps, PV^γ=const. (Note that $V_4 = V_1$ and $V_2 = V_3$).

$$P_4 V_1^\gamma = P_3 V_2^\gamma \quad \Rightarrow \quad P_4 = P_3 \left(\frac{V_2}{V_1}\right)^\gamma ,$$

and

$$P_1 V_1^\gamma = P_2 V_2^\gamma \quad \Rightarrow \quad P_1 = P_2 \left(\frac{V_2}{V_1}\right)^\gamma ,$$

to get

$$P_4 - P_1 = (P_3 - P_2) \left(\frac{V_2}{V_1}\right)^\gamma .$$

From Eq. (2) the efficiency becomes

$$e = 1 - \frac{Q_c}{Q_h} = 1 - \left(\frac{V_2}{V_1}\right)^{\gamma-1} ,$$

which is the same as Eq. (4.10) in *Schroeder*.