Problem 2.25

The net distance walked becomes

$$d = (N_\uparrow - N_\downarrow)\ell = [(N/2 + x) - (N/2 - x)]\ell = 2x\ell,$$

where ℓ is the step length.

Part (a)

The maximum Ω is when $N_\uparrow = N_\downarrow$.

Part (b)

From problem 2.24 we have

$$\Omega(x) = \Omega_{\text{max}} e^{-2x^2/N},$$

and we conclude that the probability for a walk with x is

$$P(x) = Ce^{-2x^2/N},$$

where normalization demands that

$$\int P(x)dx = C \int e^{-2x^2/N} dx = 1.$$

A measure of the typical distance is the root-mean-square distance

$$\sqrt{\langle d^2 \rangle} = 2\ell\sqrt{\langle x^2 \rangle},$$

where

$$\begin{aligned} \langle x^2 \rangle &= \int_{-\infty}^{\infty} P(x)x^2 dx = C \int x^2 e^{-2x^2/N} dx \\ &= C\left[-\frac{xN}{4} e^{-2x^2/N}\right]_{-\infty}^{\infty} + \frac{CN}{4} \int e^{-2x^2/N} dx = \frac{N}{4}, \end{aligned}$$

which gives

$$\sqrt{\langle d^2 \rangle} = \ell\sqrt{N}.$$

After $N = 10000$ steps the walker will typically be at the distance of $\sqrt{N} = 100$ steps from the starting point.