Problem 1.70

In analogy with the thermal conductivity, derive an approximate formula for the diffusion coefficient, D in

$$J_D = -D\frac{\partial n}{\partial x}, \tag{1}$$

of an ideal gas in terms of the mean free path and the average thermal speed.

Solution

(By V. Bychkov. Geometry as in Fig. 1.18.)

Consider two partitions, 1 and 2, with density $n_1 = N_1/(A\ell)$ and similarly for n_2. Since the witdh of the box is equal to ℓ (and we assume that all motion is along the x direction only) all the particles in the respective boxes will leave its box during the time $t_{\rm coll}$, half of the particles in box 1 will end up in box 2. The net particle flow crossing the wall between the partitions is then

$$\Delta N = \frac{1}{2}(N_1 - N_2) = \frac{1}{2}A\ell(n_1 - n_2) = -\frac{1}{2}A\ell^2\frac{\partial n}{\partial x}.$$

Since this flow takes place during time $t_{\rm coll}$ the flux—the number of particles per unit area and unit time—is

$$J_D = \frac{1}{A}\frac{\Delta N}{t_{\rm coll}} = -\frac{1}{2}\frac{\ell^2}{t_{\rm coll}}\frac{\partial n}{\partial x}.$$

With

$$v_{\rm th} = \ell/t_{\rm coll},$$

and Eq. (1) we identify

$$D = \frac{1}{2}\ell v_{\rm th}.$$