1 Outdoor refrigerator

Some people put the freezer in an outhouse with outdoor temperature to save energy. Estimate the saving (in percent) by moving the freezer outdoors if we assume optimal efficiency, indoor temperature $T_{\text{in}} = 20°\text{C}$, outdoor temperature $T_{\text{out}} = 0°\text{C}$, and the desired temperature in the freezer, $T_c = -20°\text{C}$. (4p)

(It might well be that some freezers don't work well at such conditions, but that need not concern us here.)

Solution: First, introduce T_h for the temperature of the environment, which will be either T_{in} or T_{out}. Let Q_c be heat entering the system and Q_h be heat leaving the system (which thus doesn't obey the sign convention).

For optimal efficiency we have that entropy entering and leaving the system are equal:

$$\frac{Q_h}{T_h} = \frac{Q_c}{T_c} \quad \Rightarrow \quad Q_h = \frac{T_h}{T_c} Q_c,$$

and then $W = Q_h - Q_c = (T_h/T_c - 1)Q_c$.

The amount of heat that enters the system is given by the temperature difference

$$Q_c = K(T_h - T_c),$$

where K is a constant. Taken together these two relations give

$$W = \frac{K}{T_c}(T_h - T_c)^2,$$

and for the two different situations we find

$$\frac{W_{\text{out}}}{W_{\text{in}}} = \left(\frac{T_{\text{out}} - T_c}{T_{\text{in}} - T_c}\right)^2 = \left(\frac{20}{40}\right)^2 = 0.25.$$

The saving is 75%.