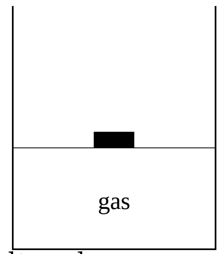
2 **Expanding ideal gas** version from 170109

A diatomic ideal gas is enclosed in a cylinder by a frictionless piston as shown in the figure. Due to a load of mass m, the initial equilibrium state of the gas is $p_i = 7.0 \times 10^5$ Pa, $V_i = 2.0$ dm^3, and $T_i = 27°$C. The gas is expanded quasistatically to the final volume $V_f = 2V_i$ in two different ways:



a) By a slow supply of heat Q, while the load is unaltered, or

b) by a slow elimination of small pieces of the load in such a way that pV^γ is kept constant during the process.

Calculate the final temperature of the gas, and the works performed by the gas, in the two processes.

Solution:

a) This is an isobaric processs i.e. a process with constant pressure. One then has

$$p = \frac{Nk_BT}{V} \quad \Rightarrow \quad T_f = \frac{V_f}{V_i}T_i = 600K.$$

The work performed by the gas is

$$-W = \int_{V_i}^{V_f} pdV = p(V_f - V_i) = pV_i = 7 \times 10^5 \ \cdot \ 2 \times 10^{-3} = 1.4\text{kJ}.$$

b) In this process the pressure is changing, $pV^\gamma = p_iV_i^\gamma$. From the ideal gas law, $T \propto pV$ we have

$$TV^{\gamma-1} = T_iV_i^{\gamma-1},$$

which gives (with $\gamma = 1.4$ for a diatomic gas)

$$T_f = T_i\left(\frac{V_i}{V_f}\right)^{\gamma-1} = 300\ 0.5^{0.4} = 227\text{K}.$$

With $U = (f/2)Nk_BT$, $\Delta U = W + Q$, and $Q = 0$ one finds (after using $p_iV_i = Nk_BT_i$)

$$-W = -\Delta U = \frac{f}{2}Nk_B(T_i - T_f) = \frac{f}{2}p_iV_i(1 - T_f/T_i) = \frac{5}{2}p_iV_i(1 - 0.5^{0.4}) = 847\text{J}.$$

(Using the approximate $1 - 227/300$ gives $-W = 852$J which was given in the solutions.)