Master equation solution of a two-state system

(a) Note that $w_0 + w_1 = 1$. dw_0/dt is the probability of state 0 times the probability of a change to state 1 minus the probability of state 1 times the probability of a change to state 0.

$$\frac{dw_0}{dt} = R_0(1 - w_0) - R_0 w_0 e^{-\beta(E_1 - E_0)} \quad (1)$$

$$= R_0 - w_0 \left(R_0 + R_0 e^{-\beta(E_1 - E_0)} \right) \quad (2)$$

(b) Integration (multiply by integration factor) gives

$$w_0(t) = c_1 e^{-c_2 t} + \frac{1}{1 + e^{-\beta(E_1 - E_0)}} \quad (3)$$

where

$$c_0 = R_0 + R_0 e^{-\beta(E_1 - E_0)} \quad (4)$$

and

$$c_1 = \frac{-1}{1 + e^{-\beta(E_1 - E_0)}}. \quad (5)$$

This gives $w_0(\infty) = 1/(1 + e^{-\beta(E_1 - E_0)})$ and $w_1(\infty) = e^{-\beta(E_1 - E_0)}/(1 + e^{-\beta(E_1 - E_0)})$ and thus the Boltzmann distribution is fulfilled as $w_1/w_0 = e^{-\beta E_1}/e^{-\beta E_0}$.

Conserved order parameter Ising model

(b) Any two configurations will differ at an even number of sites $s_1, \ldots, s_N$. There is a finite chance for the chosen sites (in step 2) to be (s_1, s_2), (s_3, s_4), $\ldots$, (s_{N_1}, s_N). So the system is ergodic.

(d) Follows the calculations for the Metropolis algorithm exactly.