3.7.3 Fishery model

The equation

$$\dot{N} = rN(1 - N/K) - H,$$

provides an extremely simple model of a fishery. In the absence of fishing, the populatin is assumed to grow logistically. The effects of fishing are modeled by the term $-H$, which sys that fish are caught or “arvested” at a constant rate $H > 0$, independent of their population N.

a) Show that the system can be rewritten i dimensionless form as

$$\frac{dx}{d\tau} = x(1 - x) - h,$$

for suitably defined dimensionless quantities x, τ, and h.

Solution: Use $N = Kx$, $dt = d\tau/r$, and $H = rKh$ to get

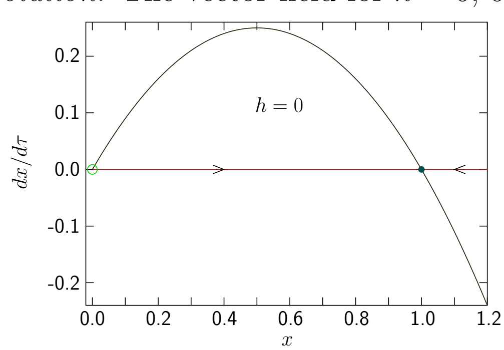
$$rK\frac{dx}{d\tau} = rKx(1 - x) - rKh.$$

After dividing by rK we obtain the desired expression

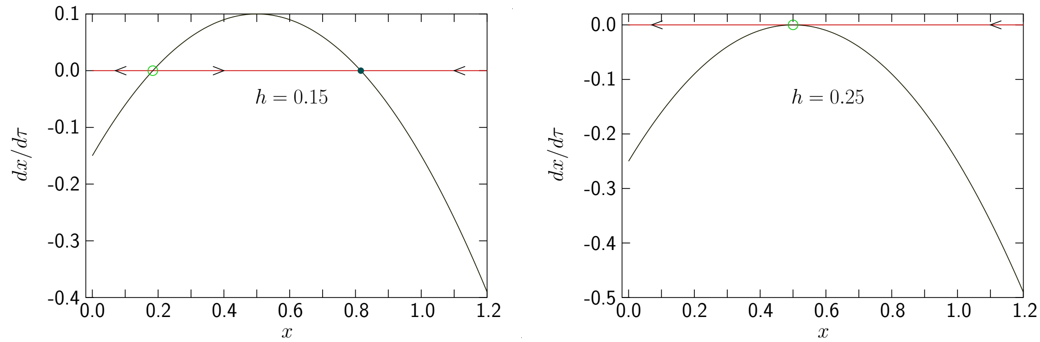
$$\frac{dx}{d\tau} = x(1 - x) - h.$$

b) Plot the vector field for different values of h.

Solution: The vector field for $h = 0$, 0.15, and 0.25:



With $h = 0$, no fishing we have the standard logistic map: Unstable fixed point at $x = 0$. Stable fixed point at $x = 1$.



With $h > 0$ there is a value below which the fish would go to extinction.

c) Show that a bifurcation occurs at a certain value h_c, and classify this bifurcation.

Solution: There is a saddle-node bifurcation at $h = 0.25$.

d) Discuss the long-term behavior of the fish population for $h < h_c$ and $h > h_c$, and give the biological interpretation in each case.

Solution: Always extinction for $h > h_c$. Lower stable fish population (compared to the case with $h = 0$) for $h < h_c$ but also a possibility of extinction if the population (for some reason) temporarily goes down to values below the unstable fixed point.