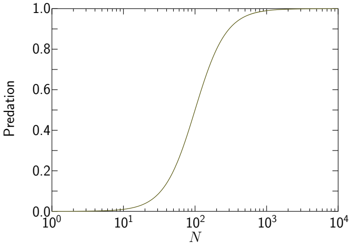
3.7 Insect outbreak

The dynamics is given by

$$\frac{dN}{dt} = r_B N \left(1 - \frac{N}{K_B}\right) - p(N),$$

where $p(N)$ represents predation by birds.



We now specialize to the following form for predation:

$$p(N) = \frac{BN^2}{A^2 + N^2}$$

which has a change from low to high predation at an approximate threshold value $N_c = A$. The dynamics now becomes

$$\frac{dN}{dt} = r_B N \left(1 - \frac{N}{K_B}\right) - \frac{BN^2}{A^2 + N^2}.$$

3.7 Insect outbreak. . . Nondimensionalisation

With four parameters, r_B, K_B, B, and A and it is difficult to analyze the model. To simplify things we introduce the dimensionless quantities

$$u = \frac{N}{A}, \quad r = \frac{Ar_B}{B}, \quad q = \frac{K_B}{A}, \quad \tau = \frac{Bt}{A},$$

which leads to the equation

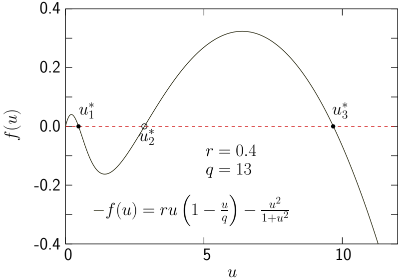
$$\frac{du}{d\tau} = ru\left(1 - \frac{u}{q}\right) - \frac{u^2}{1+u^2} = f(u; r, q).$$

- Two parameters r and q which are pure numbers.
- The time scale is also changed.
- (Also other possible ways to make things dimensionless.)

3.7 Insect outbreak... Fixed points

- The fixed points are where $f(u) = 0$.
- Three nontrivial solutions: u_1^*, u_2^*, u_3^*.
- Stable of unstable? Examine $f'(u^*)$!

  - Here $du/d\tau = f(u; r, q)$.
  - Consider the sign of $\partial f/\partial u$.
    - ★ u_1: $f'(u_1) < 0$ — stable.
    - ★ u_2: $f'(u_2) > 0$ — unstable.
    - ★ u_3: $f'(u_3) < 0$ — stable.



How will things change with the parameters, r and q?

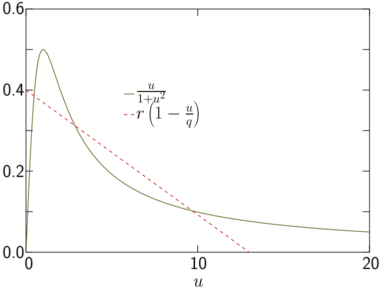
3.7 Insect outbreak. . . Innovative graphical solution

The steady states are solutions of

$$f(u;r,q) = 0 \Rightarrow ru\left(1-\frac{u}{q}\right) = \frac{u^2}{1+u^2}.$$

Look for the solutions to

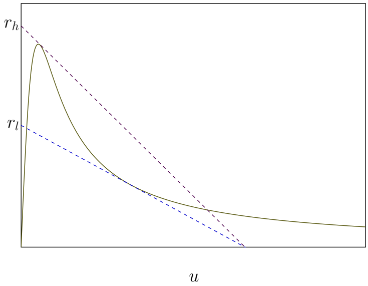
$$r\left(1-\frac{u}{q}\right) = \frac{u}{1+u^2}.$$



- The right hand side is a curve with a non-trivial shape, independent of r and q.
- The left hand side depends on the parameters but is simply a straight line from $(0, r)$ to $(q, 0)$.
- The solutions are given by the intersections of these curves. Different r and q give different straight lines and either one or three solutions.

3.7 Insect outbreak... The effect of changing the parameters

Three solutions for $r_l \leq r \leq r_h$. Otherwise only one solution.



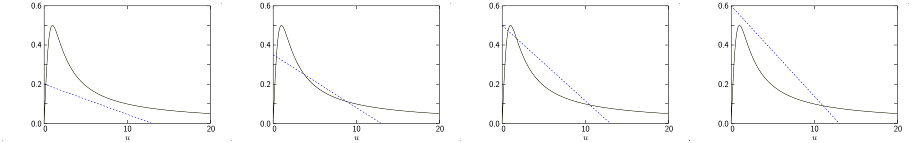
What is the effect of a gradual change of the parameter r?

It turns out that we get a hysteretical behavior.

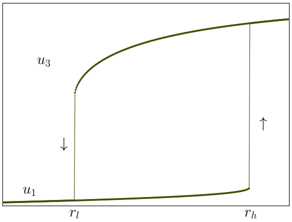
(It could be that r changes gradually because of changes in the environment.)

3.7 Insect outbreak... The effect of changing the parameters

Consider a gradual change of r from a small value to $r > r_h$ and back again!



- When $r > r_h$ the fixed point u_1^* disappears. The system jumps to u_3^*.
- When $r < r_h$ not much happens; the system remains at u_3^*.
- When $r < r_l$ the fixed point u_3^* disappears and the system jumps back to u_1^*.



3.7 Insect outbreak. . . Concern for the environment

The behavior above show the mechanism behind a tipping point:

- Things first just change gradually and slowly,
- When a parameter exceeds some critical value the system jumps to a different fixed point.
- Even if the parameter could be lowered below that critical value, the system could be stuck at this new fixed point.

Next lecture (Friday)

5. Linear systems with $n = 2$.
   - harmonic oscillator,
   - uncoupled equations,
   - classifications of linear systems,

Compare with the classification:

	$n = 1$	$n = 2$	$n \geq 3$	$n \gg 1$	continuum
lin-ear	growth, decay or equilibrium	oscillations		solid state physics	elasticity, wave eqs
non-lin-ear	Fixed points, bifurcations	pendulum, limit cycles	chaos, strange attractors	*research problems of today*	